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An International Review of Spectroscopy and Astronomical Physics

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THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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FIVE LITHIUM LINES AND THEIR MAGNETIC SEPARATION

By NORTON A. KENT

INTRODUCTION

In the Zeeman effect given by narrow, double, and complex lines Michelson,[1] Gmelin,[2] and Wendt[3] observed assymmetries. The interest involved in these facts arose from the knowledge that the complicated magnetic separation of series doublets and triplets was dependent upon the close relationship of the vibrating systems within the atom (Paschen,[4] 1909), and further from the proof that narrow series doublets and triplets suffered a magnetic change, the final result of which was a single normal triplet (Paschen and Back[5]). Examples suitable for a study of these changes, Na λλ 3303 and 2853, were treated by Back[6] and Fortrat[7] respectively.

Zeeman[8] had shown Li λ 6708 a double line and Back had succeeded, at 29,800 gausses, in obtaining with this and other lithium lines, as a final effect, a single normal triplet.

[1] *Astrophysical Journal*, **8**, 46, 1898.
[2] Dissertation, Tübingen, 1909.
[3] Dissertation, Tübingen, 1911.
[4] *Annalen der Physik*, **30**, 746, 1909.
[5] *Ibid.*, **39**, 897, 1912; **40**, 960, 1913.
[6] Anhang., *Annalen der Physik*, **39**, 926, 1912; also an unpublished paper on the same line.
[7] *Comptes rendus*, **156**, 1607, 1913, and **157**, 639, 1913.
[8] *Proc. Royal Acad. Amsterdam*, May 30, 1913; September 3, 1913.

Moreover, mainly upon the work of Paschen and Back, as a basis, Voigt[1] had formulated a mathematical theory of the Zeeman effect for close pairs homologous to Na λ 3303, the final result predicted being a quintuplet with simple parallel component and two perpendicular components, each a close pair. In this work he dealt particularly with unpublished experiments of Back on Na λ 3303.

As a logical sequence, then, of these results and of their theoretical explanation and because the magnetic change of a doublet such as D_1 and D_2 of sodium is a simple one and further had been treated theoretically by Voigt, it was suggested by Professor Paschen that the writer make a study of Li λ 6708. As the work progressed it became evident that other lithium lines could be obtained and these too were briefly studied.

APPARATUS AND METHOD OF PROCEDURE

1. The spectroscopic system consisted of a Hilger echelon and constant-deviation prism placed in juxtaposition. The echelon constants were: number of plates, 33; thickness, 9.487 mm; step, 1.01 mm; aperture, 39×34.3 mm. The collimator and telescope lenses were Zeiss achromats of 5-cm aperture and 70-cm focal length. Between the telescope objective and the photographic plate was inserted a simple, double-concave lens, so situated that the resultant magnification was approximately fourfold.[2] This arrangement was used in all the photographic and most of the visual work. This optical system gave about 20 times as intense a spectrum as the second order of a 21-foot concave grating, and a dispersion and resolving power on the average, in the visible region, equivalent to the fourth and fifth orders respectively. During exposures a delicate thermometer, inserted in the case covering the echelon, was held constant in temperature to at least 0.3°, and usually to less than 0.1° C.

The single-order position was generally used—obtained by altering either the temperature of the room or the angle of the

[1] *Annalen der Physik*, **41**, 403, 1913; **42**, 210, 1913.

[2] Twofold magnification was also used. See Table II.

instrument. The difference in scale between the two positions was less than the error of measurement of the plates.

2. As sources of light there were used:

a) Vacuum tubes of various forms into which were introduced fused lithium chloride,[1] ground fine by mortar and pestle. In order of trial these tubes are represented in Fig. 1. The walls of (*a*) soon blacken and become too opaque for use "side on," while "end on" in the magnetic field a poor image is given. Form (*b*), in which the cathode is an aluminium cup filled with the salt, serves better with no field, but with field the image is lost entirely; (*c*) works well, but the salt is soon exhausted and the process of filling laborious; (*d*) serves excellently.[2] The first tube of this kind, of ordinary glass was short lived: quartz proved very durable. Nearly all the published results were obtained with Heraeus quartz tubes such as this.

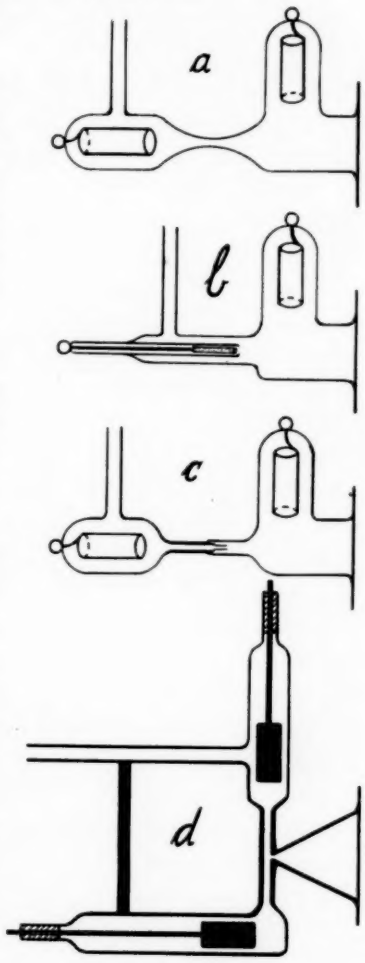

FIG. 1

The lithium salt, introduced through the window with the capillary held horizontal, was fused to the walls of the tube with a small flame. Such a tube lasts many hours, and, when the stream of ions has cut the quartz, the capillary may be cleaned and re-fused with an oxygen flame.

[1] Lithium nitrate and lithium oxide were also tried but found unsuitable.

[2] A similar form was used by Runge and Paschen: see *Astrophysical Journal*, 15, 238, 1902.

The tubes were excited by a Klingelfuss induction coil, the primary current varying from 1 to 6 amperes, generally being about 1.5, and the frequency from 45 to 60 per second. With high fields larger currents and higher gas pressures were necessary. To the evacuating system were added two important accessories—a large bulb (0.5 liter capacity) to prevent rapid decrease of pressure by self-evacuation, and an apparatus for generating and drying hydrogen. The strength and character of the lithium lines are very sensitive to changes in pressure and kind of gas.[1] The tube was washed repeatedly with dry hydrogen before and during the discharge. The most favorable conditions were then found by pumping out the hydrogen, observing the tube meanwhile with a pocket spectroscope. During long exposures this process was several times repeated.

b) The spark in air between carbon terminals soaked in a fused alloy of lithium and sodium. Capacity and inductance were inserted in the spark circuit.

c) Trembleur *in vacuo*. A lithium-cadmium alloy of any desired strength was prepared electrolytically by running a known current for a given time through melted cadmium covered with fused lithium chloride, the negative terminal being an iron wire inserted in the cadmium and insulated above where it passed through the lithium chloride; and the positive being a carbon rod, touching only the lithium salt. Pieces of the alloy were then later melted upon brass or manganin terminals. The vacuum used was generally about 5 mm. The peculiar form of trembleur employed was due to Dr. Back[2] with whom the trembleur-echelon part of the present work was performed. His able assistance, both in this regard and in other minor ways, the writer sincerely appreciates. Certain high-field plates were also taken by him alone, with the trembleur and the 21-foot Rowland grating.

3. As electromagnets there were used a small instrument; a large magnet made at the Institute; and, with the trembleur, a large Weiss instrument. The values of the fields used with the

[1] Air and oxygen were also tried, the latter proving especially disadvantageous.

[2] A description of this instrument will shortly be published by Dr. Back in the *Annalen der Physik*.

first two were measured by a galvanometer calibrated by standard coil and exploring spool. Errors of determination are certainly less than 2 per cent. With the Weiss instrument the fields were measured by the known Zeeman separation of certain cadmium lines, these photographs being taken by Dr. Back with the trembleur and grating under conditions as nearly as possible those obtaining in the echelon exposures. Moreover, as Cd λ 6439 appeared on the echelon plates with Li λ 6708, measurements thereon served as a check, although the images were somewhat overexposed. The two sets of measurements agree to about 1 per cent.

4. The plates used were Wratten and Wainwright, "Panchromatic B, backed" for λλ 8127, 6708[1] and 6104; Hauff "Orthochromatic" for λ 4972; and Hauff "Extra-rapid" for λ 4602. For development, glycine in rather weak solution was used.

5. The plates were measured on a Zeiss comparator, four settings on each line, red-right and four, red-left. The error of measurement, varying according to the character of the plate and of the line, was on the average, about 0.003 A on vacuum-tube plates, and less than 0.006 A on the trembleur.

In determining the separation of the doublet components, the values of the constant k for the echelon were calculated from the measurements of Gmelin.[2] The provisional value of the wave-length difference is: $\Delta\lambda_p = k\frac{dn}{dn'}$, where dn is the number of divisions of the comparator between the two lines of the pair and dn' that between the same component in adjacent orders. The prism scale was determined from lines of known wave-length, generally those of a helium tube, and the ratio of the prism dispersion to that of the echelon (varying from 1 to 3.4 per cent) was then applied as a positive[3] correction to $\Delta\lambda_p$.

6. To separate the parallel and perpendicular components, a double-image calc-spar crystal (generally oriented so that the two images lay one above the other) was set before the slit.

[1] These plates, stained with dicyanine, were tried with λ 6708, but proved inferior. The stain was indispensable however with λ 8127.

[2] Inaugural Dissertation, Tübingen, 1909.

[3] The direction of longer wave-lengths in the echelon spectrum (determined by the known position of the satellite of He λ 5875.87) lay toward that of shorter as given by the prism.

RESULTS

The results obtained comprise a study of (*a*) five lines of the lithium spectrum with no magnetic field, and (*b*) their magnetic separation.

(*a*) Of these five lines Zeeman[1] had already developed the doublet λ 6708 as an absorption line and determined the separation of its components as 0.144 A. The other four had not previously been shown double. The writer has obtained all as narrow emission lines and found them double. Tables I, II, and III present the results. The sources were vacuum tubes of form (*c*) (Fig. 1) for plate 13 and form (*d*) for the others. Professor Paschen very kindly measured plates 80 and 75. His values are given under P (Table I), the writer's under K. The average deviation from the mean of measurements given in these two tables is about 0.001 A, or 1 per cent. The second series was taken to test the validity of the values obtained in the first. Table III presents the weighted means of both series and, in addition, measurements made upon certain λ 8127 plates taken by Professor Paschen and Mr. Ignatieff at zero and $3\frac{1}{4}$-fold magnification.

It seems, then, that the frequency variations must be real. There is substantial agreement between the values obtained for the second subordinate series (.340, .336, .339), but those for the first subordinate (.309, .328) differ both from each other and from those of the first series. Zeeman has already stated that according to known series laws the frequency difference for the red lithium line is too small compared with that of the other alkalies. The same was noted before by Popow[2] for certain beryllium lines.

For reproductions of these pairs see Plate XV, A. The enlargements (negatives) therein shown are, unless otherwise stated, 4.5 the size of the photographed image, or 18 times that given without the divergent lens. The scale is 1 A = 1 cm. Longer wavelengths lie to the left. In section A in order from left to right are: λ 6708. The very strong single line is He λ 6678. Note the satellite (faint) between it and the doublet; λλ 6104, 4972, and 4602—

[1] *Proc. Royal Acad. Amsterdam*, September 3, 1913.

[2] *Verhandlungen der Schweizerischen Naturforschenden Gesellschaft*, September 9, 1913.

PLATE XV

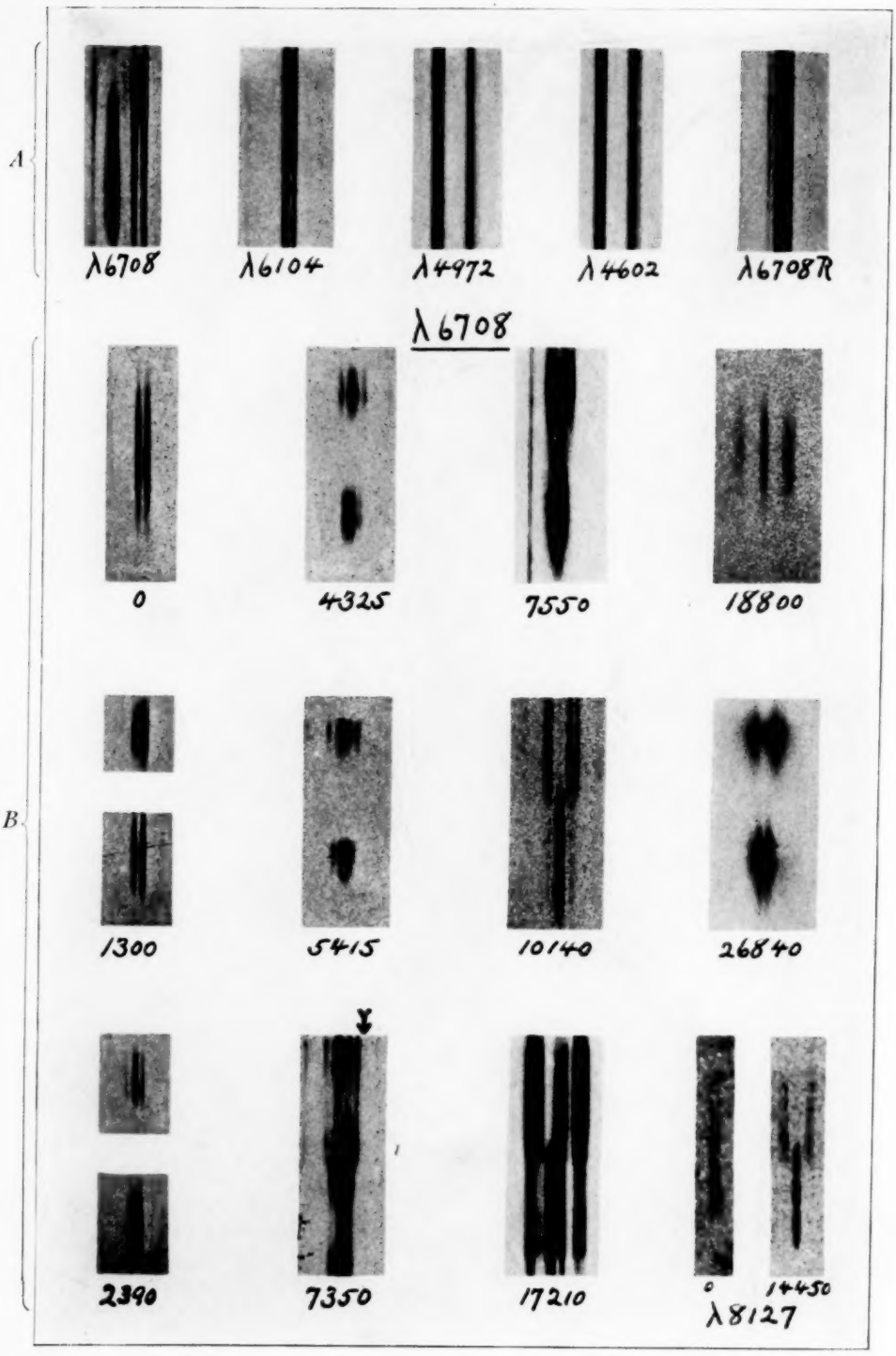
A
λ6708
λ6104
λ4972
λ4602
λ6708R
λ6708
B
0
4325
7550
18800
1300
5415
10140
26840
2390
7350
17210
0
14450
λ8127

TABLE I

First Series, Fourfold Magnification

Number of Plate	Time of Exposure in Minutes*	Character of Plate	Wave-Length of Line λ, in A	Series Classification†	Provisional Value of Wave-Length Difference $\Delta\lambda\rho$ in A		Correction for Prism Dispersion in A		Final Value of Wave-Length Difference $\Delta\lambda$, in A			Frequency Difference $\frac{\Delta\lambda}{\lambda^2}=\Delta\nu$, in cm.$^{-1}$		
					K		K		K		Mean	K		
13*b*	8	Excellent	6708.2	$1.5s-2p_i$	.1485		.0019		.1504‡		.151	.336		
					.1478		.0019		.1497§					
13*a* and *b*	5 and 8	Excellent			.1482		.0019		.1501¶					
84	11	Excellent			.1508		.0021		.1529					
					P	K	P	K	P	K	Mean	P	K	Mean
80	12	Excellent	6103.77	$2p-3d$	.1105	.1124	.0033	.0015	.1138	.1139	.114	.3054	.3057	.306
75	45	Very weak	4972.11	$2p_i-3.5s$	.0809	.0839	.0016	.0017	.0825	.0855	.084	.3337	.3458	.339
					K		K		K			K		
77	5	Excellent	4602.37	$2p_i-4d$	.0674		.0017		.069			.326		

* The exposure times here given are typical of those of the entire investigation, the limits being about as here shown.

† See F. Paschen, *Jahrbuch der Radioaktivität und Elektronik*, 1911, Bd. 8, Heft 1, p. 174.

‡ First set of measurements. § Second set. ¶ Third set; values obtained from 13 *a* and *b* combined.

TABLE II

SECOND SERIES, TWOFOLD MAGNIFICATION

Number of Plate	Time of Exposure in Minutes	Character of Plate	λ	$\Delta\lambda_p$ in A	Correction for Prism Dispersion in A	$\Delta\lambda$ in A	Mean $\Delta\lambda$ in A	$\Delta\nu$ in cm^{-1}
87	5	Excellent	6103.77	.1129	.0023	.1152	.115	.310
89*a*	3	Excellent		.1162	24	.1186		
b	3	"		.1134	23	.1157		
91*a*	3	Fair		.1131	18	.1149		
b	3	"		.1135	18	.1153		
93*a*	10	Fair		.1112	16	.1128		
100	55	Excellent	4972.11	.0812	23	.0835	.084	.339
95*a*	4	Excellent	4602.37	.0671	21	.0691	.070	.329
b	4	"		.0661	22	.0683		
97*a*	4	Excellent		.0690	23	.0713		
b	5	"		.0678	.0023	.0701		

TABLE III

λ	MEANS OF SERIES				WEIGHTS ASSIGNED TO VALUES IN SERIES		WEIGHTED MEANS	
	I		II					
	$\Delta\lambda$	$\Delta\nu$	$\Delta\lambda$	$\Delta\nu$	I	II	$\Delta\lambda$ in A	$\Delta\nu$ in cm^{-1}
8127.1*....	—	—	—	—	—	—	.225	.340
6708.2.....	.151	.336	—	—	—	—	.151	.336
6103.77....	.114	.306	.115	.310	1	6	.115	.309
4972.11....	.084	.339	.084	.339	1	1	.084	.339
4602.37....	.069	.326	.070	.329	1	4	.070	.328

* Series Classification, $2p_1$–$2.5s$.

in double order condition; λ 6708, Li_1 and Li_2 *reversed* and the satellite (unreversed) 0.146 A from Li_1 or 0.222 A from the center of symmetry. λ 8127 is shown in the next to the last cut of Plate XV, section B, and this cut and the one following are tenfold enlargements of negatives made without any divergent lens. The stronger line in λ 6708 lies toward the violet; in the other four to the red. In confirming this statement the reader must bear in mind that the relative intensities as photographed depend upon the position of the echelon, whether accurately or only approximately in single or double order. No real change in the relative intensity of the lines of λ 6708 was noted. Zeeman found that with increased vapor density the violet component was less intense than the red. This may have been due to the coming up of the weak satellite beyond the red component. However, Zeeman does not offer this explanation, although he does note that "with still greater density new absorption lines appear in the vicinity of the lithium pair."

(*b*) The magnetic resolution was studied both photographically and visually for λ 6708 in detail, briefly for λλ 8127 (by photograph only) and 6104; and briefly, visually, for λλ 4972 and 4602. In the visual work Professor Paschen, Dr. Back, and the writer all shared, the greater part of the observing, however, being done by Professor Paschen who, moreover, together with Mr. Ignatieff, at the cost of much labor, very kindly made the previously mentioned exposures on λ 8127 and certain others also on λ 6708 to obtain plates unquestionably free from reversal. This was accomplished with λ 6708 at low fields and incidentally the sixth perpendicular component was found present although very weak (see p. 350). With a field of 12,000, reversal became so hard to avoid that resort was had to λ 8127.

The advantage of visual observation with λ 6708 lay in the fact that the presence of reversal could thus easily be detected. Both the photographic and visual results here given were, unless otherwise stated, obtained with quartz vacuum tubes of form (*d*), Fig. 1. Statements based on the visual work are indicated by (v).

λ 8127: See p. 353.

λ 6708: This doublet exhibits a magnetic separation shown in Plate XV, B, and represented schematically in Fig. 2.

In the plate the perpendicular structure lies above, the parallel below. The values of the fields in gausses are given beneath each negative. The exposure at 18,800 gausses was made without the calc-spar crystal. Concerning Figs. 2, 3, and 4 certain general remarks may here be made.

Fig. 2: This schematic representation is composed of sixteen sections, the first representing the theoretical structure and approximate relative intensity of D_1 and D_2 of sodium in an imaginary field and the remaining fifteen that shown by Li λ 6708 from 1300 to 44,200 gausses. In these last sections the positions, in zero field, of the components of the lithium line (Li_1 and Li_2, section I), and their geometrical mean are shown by short lines at the top of each section. The series of lines following show successively (1) the components of the theoretical structure perpendicular to the lines of force, according to Preston's rule, (2) the observed perpendicular components, (3) the Preston theoretical parallel, and (4) the observed parallel components (series 3 and 4, broken lines). In all cases the theoretical components are designated by letters (as in sections I and II), the observed by numbers (section II). At XI, however, to as much of the theoretical structure of D_1 and D_2 as still falls within the limits of the plot is added that of a normal triplet, components I, II, III. This is plotted to a point, *p*, 0.020 A to the violet of the geometrical center of Li_1 and Li_2, the mean position of the line 8, 9 as measured on eleven plates.

The approximate relative intensities are represented by the lengths of the lines, the scale of the theoretical being smaller however than that of the observed.

The positions of the observed lines of sections II, III, VI, VIII, IX, and XIV are located arbitrarily as a whole (each line, of course, retaining its measured distance from its neighbors) without regard to a reference line, while those of IV, V, and VII are set accurately by λ 6563 of hydrogen as a standard, those of X to XIII by λ 6678 of helium and those of XV and XVI by λ 6439 of cadmium.

Sections II to XIV show vacuum-tube plates; XV and XVI trembleur plates.

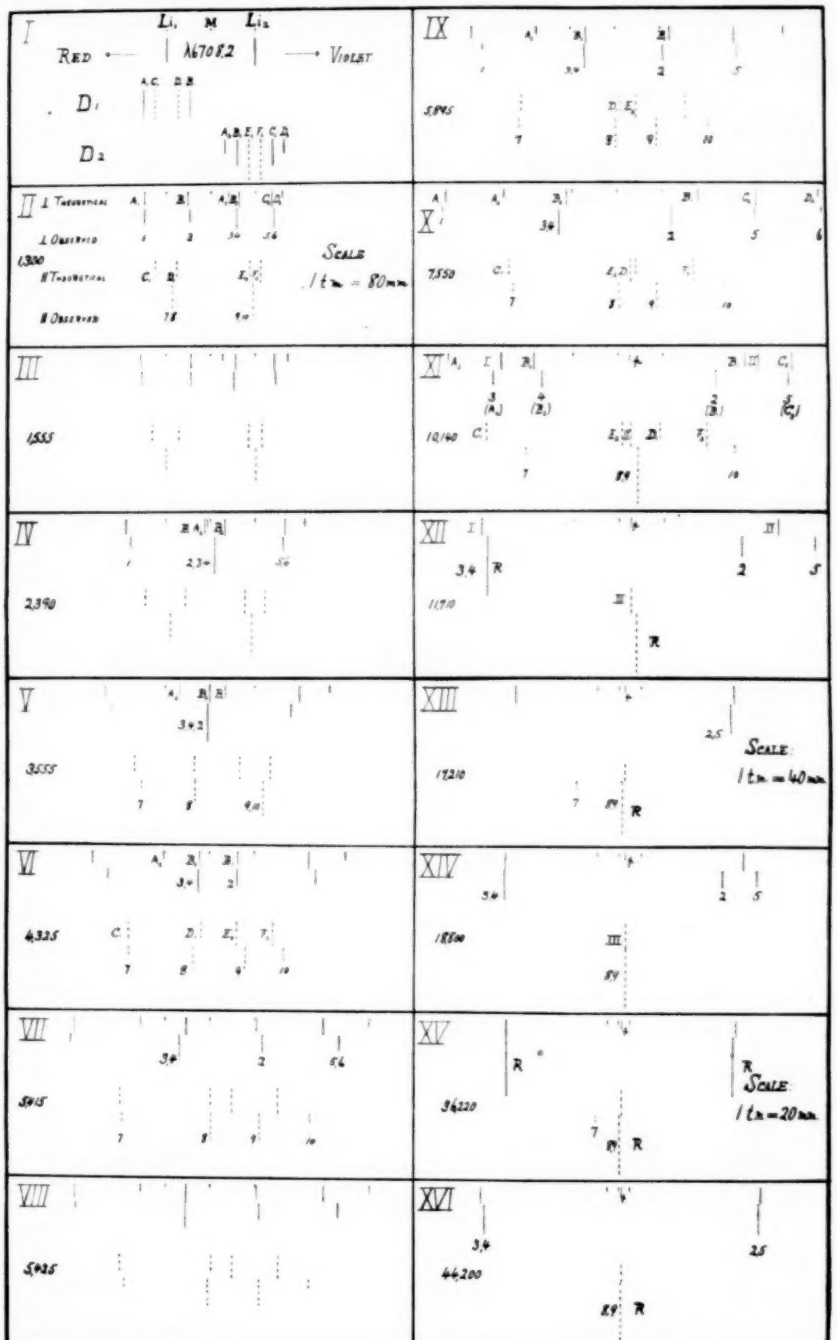
I
Li1 M Li2
RED
λ6708.2
VIOLET
D1
D2
II
SCALE
I tm = 80 mm.
III
IV
V
VI
VII
VIII
IX
X
XI
XII
XIII
SCALE
I tm = 40 mm.
XIV
XV
SCALE
I tm = 20 mm.
XVI

FIG. 2

The wave-length scales are: sections II to XII, 1 A = 80 mm; XIII and XIV, 40 mm; and XV and XVI, 20 mm. The greatest error of measurement is not more than 1, 0.5 and 0.25 mm for the three scales, and the average error is but a fourth as great, or 0.003 A.

If a plate does not show a certain line we must not conclude that it does not exist; it may be either weak or unresolved.

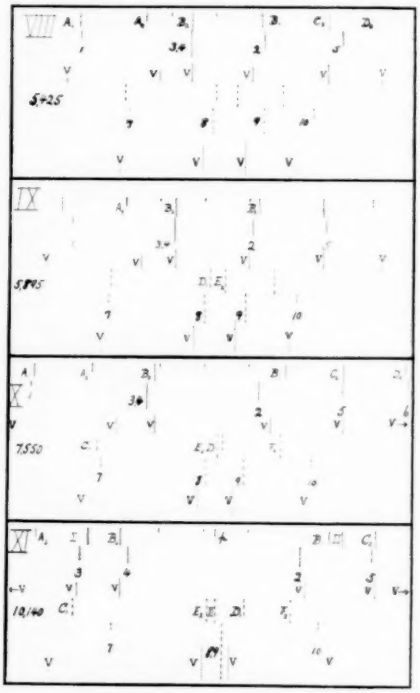

FIG. 3

In X, lines 1 (too weak to measure) and 6 (not visible) are approximately placed by data from another plate at a slightly different field. The same statement holds for lines 7 and 10 in XI.

In XII to XVI, *R* indicates a measurement made to the center of reversal. The lines are here drawn full length.

In XIII the helium line interferes with 3,4 which is therefore omitted; 2,5 is a broad line.

Fig. 3, composed of sections taken from Fig. 2, shows in addition the positions and intensities of the components, marked V, of λ 6708 according to Voigt's theory (see p. 352).

In Fig. 4, for λ 6104 the same general method as that of Fig. 2 is employed.

These three sketches were made with care on a scale five times as great as the reproductions and the errors of plotting are less than those of measurement—in fact the sketches may be regarded quantitively as substitutes for tables of wave-lengths.

For λ 6708, then, experiment shows that the components *parallel* to the lines of magnetic force are at first one line each; later

Li_1 appears double at 3555 (2700 v) gausses (Fig. 2, V), and Li_2 follows at 4325 (Fig. 2, VI). Of these four lines, the two inner increase greatly in intensity but remain clearly separated to 5500 (v) and are just distinct at 8300 (v). They fuse before 10,140 (9300 v) and appear as a strong, broad line which narrows with increase of field. Reversal occurs easily with large current in the tube and is almost unavoidable with the trembleur. The two outer lines weaken continually from even as low a field as 4325 gausses and become progressively harder to measure. This loss of intensity is by no means wholly due to the echelon. However, these "residuals" both appear on a strong tube plate at 18,070 and a trembleur plate at 19,500, while the one of longer wave-length occurs on a trembleur plate at 36,220.

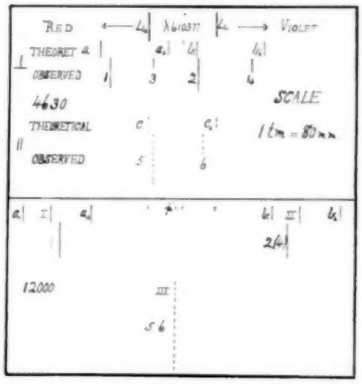

FIG. 4

The plates show certain interesting wave-length changes the first steps of which are in part similar to those noted by Dr. Back with Na λ 3303. These changes are first evident at 4325 gausses and consist of an approximate agreement with the theoretical (assuming the structure that of D_1 and D_2 of sodium) of the wave-length difference of the lines in each of the two pairs of parallel components but a progressive drawing apart of their centers of symmetry—consult Fig. 2 (fields 4325, 5415 or 5425,[1] 5895, and 7550) and Table IV. Thus, lines 7, 8 and 9, 10 preserve a distance which is approximately the theoretical, C_1-D_1 and E_2-F_2 (9,10 quite accurately so); but the centers of symmetry of the pairs should remain 0.151 A apart. They actually lie at a distance, d, as follows:

TABLE IV

Field in Gausses	d in Angstroms
4325	0.174
5415	0.198
5425	0.198
5895	0.193
7550	0.210

[1] Notice the agreement in the measurements of two plates taken under like conditions.

During this recession, the inner lines gradually draw together and finally fuse at 9300 as previously stated. By consulting Fig. 2 (sections VII to XI, XIII, and XV), it will be seen that the residual 7 maintains a constant wave-length difference (by calculation 0.183 A) from the center of 8 and 9 after a field of 5415 (note changes of scale in XIII and XV), while the residual 10 (last plotted in XI) continually recedes from 8,9, this law holding true until later it fades out entirely. Plate XV, B, 10,140, shows these residuals, 7 and 10.

Although all these wave-length changes are undoubtedly real, they are small; and the most striking change is that in the relative intensities of the four lines. Finally it is to be noted that the two inner never cross each other, as they theoretically should just previous to 7550.

The behavior of the *perpendicular* components is, however, quite different. Those of Li_1 and Li_2 not only preserve a better agreement with their theoretical position up to about 6000 gausses but cross each other (VI). Four lines are present at 4325, and five at 5415 (or 5425), one of these, line 3, not being distinct from its stronger neighbor; five are quite resolved at 6300 (v) and at 7350 six components are present; but lines 3 and 4 are not resolved and 6 is extremely weak. The residual is visible just below the arrow in 7350, Plate XV, B. This component is difficult to obtain because (*a*) it is not separated from 5 until a field of such magnitude is reached that the line lies in the outer and weaker region of the echelon diffraction pattern; (*b*) it is intrinsically weaker than its companion line, 5; and (*c*) it is decreased in intensity by the action of the field. It is, however, in its theoretical position while 3 and 4, which are first resolved at 10,140, are at a distance much less than the theoretical (A_2-B_2). This is precisely the effect observed by Dr. Back with the homologous component in sodium λ 3303.

X, 7550, shows a tendency to be noticed even in IX, namely, 2 has drawn in and away from B_1.

Thus lithium λ 6708 in low fields shows the types of the sodium D-lines in both perpendicular and parallel components.

To return to the previous discussion: The photograph at 10,140 shows four perpendicular components (a stronger plate at 12,000

shows five, line 6 having disappeared and line 1 being extremely weak). The theoretical normal triplet, components I, II, III, with p as a center is here first plotted. Observe that the distance I–II is greater than that between the centers of symmetry of 3,4 and 2,5—in other words the system has not yet reached the separation of a normal triplet.

At 18,800 (XIV), a high-field, unreversed, vacuum-tube plate, obtained with much difficulty, shows 3,4 as an almost structureless, broad line; while 2 and 5 are not yet one, but are closer than at 10,140 (0.575 as against 0.600 A). In placing these lines coincidence was assumed between 8,9 and III at p.

At 22,600 (v), 2 and 5 also have joined. The two resultant lines sharpen progressively to 44,200 and, with the trembleur, under suitable conditions, have been seen to flash out narrow and unreversed—not so sharp, however, as the single parallel component. Moreover, the three components have been observed at 31,300 gausses, with trembleur and Rowland grating, all reversed and again, narrow and unreversed.

The reproduction of Plate XV, B, 26,840 gausses, shows a reversed trembleur photograph. In the parallel structure the red residual may be noted; in the perpendicular, the reversal of the two components and the outward flaring of the ends of the lines—this because the field is uniform over only the central region. From the form of the curvature we infer that the two components are approaching each other from adjacent orders, as the lesser field shows the greater separation.

At 36,220 (XV) the triplet is normal. The two perpendicular components lie superposed, the separation having advanced to such an extent that these lines have not only receded from the field of view, but have returned from the adjacent orders. A simple calculation gives the true separation with fair accuracy.

At 44,200, 3,4, and 2,5 are broad and structureless. In calculating the separation account is taken of the fact that the perpendicular components have not only returned from adjacent orders but are again separating.

Thus, in conclusion, we may state that *in low fields the doublet* λ *6708 exhibits the types of* D_1 *and* D_2 *of sodium* and that *an increase*

of field produces a simple normal triplet. The parallel component, formed from lines 8 and 9, is well-nigh complete just beyond 8000 gausses; the red perpendicular, formed from 3 and 4, near 10,000; the violet, formed from 2 and 5, beyond 19,000; and the separation is approximately normal at this point. During this process the noteworthy facts are: (1) the enhancement of the inner components and the decrease in intensity of the outer; (2) the repulsion of the centers of symmetry of the two parallel pairs and the non-crossing over of their two inner lines; (3) the crossing over of the perpendicular component 2 of Li_1 and, later, (4), its drawing in toward the center of the perpendicular system; (5) the distorted position of component 3, it lying much nearer 4 than the theoretical; (6) the constant position of 7; and (7) the increasing distance of 10 from the center of the parallel system. The center of this triplet is shown by measurement to lie 0.020 A from the geometrical center of Li_1 and Li_2 toward Li_2—the stronger line. The foregoing numbered statements confirm Back's experiments and qualitatively all but (6) are in accord with Voigt's theory. The latter's statement[1] that the displacement of the parallel doublets is "oppositely directed" and "approximately equally great" is true to about 4 per cent, if point p be regarded as the center of the repulsion. Fig. 3 shows the good agreement between this mathematical theory and experiment. The positions and intensities of the Voigt lines (marked V) were calculated from a formula kindly given the writer by Professor Voigt and both are plotted on the same scale as the "observed" lines of Fig. 2. In VIII to X, if the parallel components were displaced about 0.020 A, the distance to p from the geometrical center of Li_1 and Li_2, the coincidence would be well-nigh exact. For all four fields shown the Voigt component for line 2 shows a shift almost the same as the experimental. Lines 3 and 4 when first resolved are at approximately the Voigt distance. On the other hand in all cases the outermost Voigt components show too great a separation, line 7 especially. That the intensities agree only approximately is largely due to the echelon.

[1] See *β*, *Annalen der Physik*, **42**, 213, 1913.

Thus for low fields Voigt's statements are in the main confirmed, but not in the final result predicted for high fields.[1] The perpendicular components have not indeed been photographed as single sharp lines because in high fields it is necessary to use large currents in the vacuum tube or employ the trembleur, in both of which cases the tendency to reverse is so strong that the parallel component, visually observed sometimes as simple, generally appears reversed and the perpendicular components often so. Thus the photograph, a composite of various states, always shows a reversed line for the parallel component and generally a broad but single line for each of the perpendicular (the coefficient of absorption of which is half as great).

On many strong, vacuum-tube plates, notably between 10,000 and 19,000 (and even one plate at 31,000), also a trembleur plate at 19,500, there is shown incomplete polarization of both parallel and perpendicular components. Plate XV, B, 10,140, shows incomplete polarization in the center of the perpendicular structure. See also, in Plate XV, 17,210 a very strong, reversed, vacuum-tube photograph which exhibits this phenomenon.[2] It is possible that this result was in part due to an incorrect orientation of the double-image crystal; but the phenomenon is in the main undoubtedly real, for, when the original component is reversed, its counterpart in the region of opposite polarization often appears unreversed.

A few exposures made with the spark in air for fields from 10,000 to 16,000 gave results in accordance with, but far inferior to, those of the vacuum tube; while the trembleur plates proved eventually of less than expected value owing to reversals.

λ 8127: At 13,300 and 14,450 gausses (see Plate XV, B, last photograph) the structure is that of λ 6708 at 10,140 but turned about, the violet perpendicular component being here completely formed while the red is still two lines—a result to be expected owing

[1] It is possible that the addition of the quadratic terms alluded to on p. 211 of Voigt's paper will both eliminate the slight disagreements with the theory at low fields and also produce the correct final result.

[2] In Fig. 2, XIII, the incomplete polarization is not shown.

to the inversion of intensities of the components of the two pairs. This makes absolutely certain the like results obtained with λ 6708, for λ 8127 does not reverse in the vacuum-tube discharge.

λ 6104: At 4630 gausses the two components of this doublet give what appear to be two triplets. The abnormalities in both positions and intensity are rendered clear by Fig. 4. Lines 1 and 4, also 5 and 6 are nearer each other than the theoretical (assuming two normal triplets). At 9000 (v) 5 and 6 have fused into a fairly sharp line, 3 has disappeared and 4 exists as a faint residual, the image at this stage resembling the helium doublet λ 5876 at 31,710 gausses.[1] At 12,000 both perpendicular components are simple lines, 4 having either faded out or drawn into 2, and the parallel component lies at p', 0.011 A from the center of symmetry. The separation 1–2 is less than I–II, that of the normal triplet. At higher fields all the lines sharpen and at 30,000 appear of equal breadth. Grating plates taken by Dr. Back at 40,000 show the separation normal.

λλ 4972 and 4602 were studied very briefly and the only statement which can be made is that high fields produce triplets.[2] These Dr. Back has lately, with grating and trembleur, shown normal at about 34,000 gausses.

In the latter part of the work an interesting fact was observed. With about 6 amperes in the primary of the coil, the tube (form *d*) being quite hot after a continuous run of over forty minutes, at rather high pressure, owing to the introduction of fresh hydrogen, λ 6708 vanished almost entirely from the spectrum and λλ 6104, 4972, and 4602 attained great brilliancy. To the naked eye the tube presented the unusual color of a bright and strong yellow.

SUMMARY

1. Lithium λλ 8127, 6708, 6104, 4972, and 4602 have been photographed as narrow emission lines, shown to be close pairs and their difference of wave-length measured.

[1] Paschen and Back, *Annalen der Physik*, **39**, 916, 1912.

[2] Hansen (*Annalen der Physik*, **43**, 25, 1914) showed λ 4972 normal at 29,800 to within 3½ per cent.

2. The frequency differences are unequal and this small variation is probably real.

3. The intensity relation of the components is the same as that of homologous lines of the doublet series of other alkali elements.

4. The magnetic resolution in low fields shows the doublet λ 6708 to be exactly the same as D_1 and D_2 of sodium, while λ 6104 gives what appears to be two triplets.

These five lines are thus true and regular series doublets and further,

5. Four of them have been shown to give in high fields simple normal triplets as was formerly shown by Dr. Back with a Rowland grating and a spark in air.

6. For λ 6708 it has been shown just which of the ten, low-field components disappear and which combine to form the three lines of the final, high-field, normal triplet (see p. 352), the change having been traced from beginning to end.

To both Dr. Back and Dr. Gerlach of the Institute hearty thanks are due for willing and able assistance given at various times; also to Mr. Ignatieff for the valuable facts added to the work from the plates taken by Professor Paschen with his assistance.

Finally, the writer must express his deep feeling of indebtedness to Professor Paschen himself, not only for the suggestion of this problem but also for his enthusiastic and invaluable co-operation, and for the opportunity of enjoying all the facilities of his wonderful laboratory.

TÜBINGEN
June 30, 1914

ON THE DISTRIBUTION OF THE ELEMENTS IN THE SOLAR ATMOSPHERE AS GIVEN BY FLASH SPECTRA[1]

By CHARLES E. ST. JOHN

The remarkable eclipse spectrum obtained by Professor Mitchell in 1905, the results of the reduction of which appeared in the *Astrophysical Journal* of December 1913, furnishes a body of observational data that lend themselves to the statistical method of discussion. In sharpness of definition this spectrum without doubt surpasses any other published, and probably no plate has been obtained more nearly at the proper epoch of the eclipse.

The general scheme of this discussion is to form numerous groups of lines on a simple plan. Each element has been considered by itself, and the lines of like solar intensity assigned to it have formed the ultimate group. This is necessary when the purpose is to bring into relief characteristics that are shown only by small differences.

The publication of a discussion of the data by Professor Mitchell in the *Astrophysical Journal* of March 1914 gives an opportunity for a consideration of the data from the point of view brought out in my papers on "Radial Motion in Sun-Spots."[2] The material used in that discussion was furnished by 506 lines, each line being measured, on an average, upon 14 plates of high dispersion, so that some 7000 differential measures were involved. The number of lines in Mitchell's Table I[3] is 2841. The present discussion is confined to the plate ending at λ 5879, and for the most part to the lines not appearing as blends in Mitchell's table, which reduces the number by approximately one-half; but what is lost in quantity is more than balanced by the precision obtained when the purity of the lines is assured. This is particularly true when characteristics of

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 88.

[2] *Mt. Wilson Contr.*, Nos. 69 and 74; *Astrophysical Journal*, **37**, 377, and **38**, 157, 1913.

[3] *Ibid.*, **38**, 424, 1913.

individual elements and lines are under investigation, as a treatment of the data *en masse* obscures the finer distinctions.

LEVEL IN GENERAL

An indication of the low level reached by Mitchell can be obtained by comparing the relative numbers of weak Fraunhofer lines obtained by different observers as shown in Table I.

TABLE I

FLASH SPECTRA LINES CORRESPONDING TO SOLAR LINES OF INTENSITY < 1

	oooo	ooo	oo	o	Total	Region	Per-centage
*Frost 1900	.....	.....	.....	5	5	350 A	.....
†Mitchell 1901	.....	.....	.....	5	5	1100	.....
‡ Evershed 1900	.....	.....	5	20	25	1500	.....
§Jewell 1900	.....	1	12	26	39	2500	.....
Mitchell 1905	20	87	220	278	605	2500	21
¶Hale and Adams without eclipse	34	46	17	8	105	180	80

* *Astrophysical Journal*, **12**, 307, 1900.
† *Ibid.*, **15**, 97, 1902.
‡ *Phil. Trans. Roy. Soc.*, **201** A, 457, 1903.
§ *Publications U.S. Naval Obs.*, **4**, Part 4, Appendix 1, 121, 1906.
¶ *Mt. Wilson Contr.*, No. 41; *Astrophysical Journal*, **30**, 222, 1909.

It is seen that 21 per cent of the lines observed by Mitchell are weaker than 1 on the Rowland scale of intensity and that the proportion reported by other eclipse observers is vanishingly small, while 80 per cent of those measured on the Hale and Adams plates are weaker than unity. It is especially noticeable that the ratio of the number observed by them to the number observed by Mitchell increases with decreased Fraunhofer intensity. In obtaining flash spectra without an eclipse, the slit of the spectrograph is held rigorously tangent to the edge of the photosphere by guiding upon a chromospheric line. This can be done with great precision and constancy by the method employed, and assures the possibility and the probability of the appearance upon the Hale-Adams plates of lines originating in the lowest levels of the solar atmosphere. From the clever method of Mitchell for fixing the critical instant of exposure, it is clear that the spectrum corresponding to practically the entire depth of the solar atmosphere could impress itself upon the

plate, and the richness of his plate in weak lines and the paucity of such lines upon the previous eclipse plates are probably accounted for by their exposures being ended too soon—that is, before the advancing moon had uncovered the whole depth of the atmosphere; or perhaps too late—that is, after the continuous spectrum of the photosphere had obliterated the weaker lines in the flash spectrum.

The above comparison appears to sustain the assumption, not only that the weaker Fraunhofer lines originate at lower depths, but also that these depths increase regularly with the decreased intensity of the solar lines.

THE BEHAVIOR OF SOME TYPICAL ELEMENTS

Iron.—From the long list of lines, 356 were selected as due only to iron, the ground of selection being that no other identifications were suggested by Mitchell; that is, they were not blends nor were they bracketed with other lines. From these, which include all the pure iron lines, the enhanced lines were excluded and the remaining lines were classified according to the Rowland intensities with the result shown in Table II.

TABLE II

FE LINES AND LEVEL AS SHOWN BY FLASH SPECTRA

	Number of Lines									
	4	19	30	55	72	49	40	28	24	25
Solar intensities..	∞	0	1	2	3	4	5	6	7–8–9 (7.8)	10–40 (16.4)
Flash intensities..	0.25	0.26	0.33	0.8	1.4	1.6	2.0	3.5	3.8	7.0
Heights in km...	275	279	288	344	369	397	425	488	590	806

The regular progression shown by the flash intensities and the mean of the heights assigned to iron lines corresponding to like Rowland intensities, as these intensities increase from ∞ to 16.4, lead to the conclusion that in a quite definite sense one is justified in saying that the lines of iron of a given intensity in the Fraunhofer spectrum extend to quite definite elevations. The regular march of the elevations with solar intensities is more plainly shown when the intensities are plotted as abscissae and the heights as ordinates. As a convenient scale, in lieu of a better, the numerical intensity

intervals are used as abscissae. Owing to the uncertainty in the Rowland intensity intervals the smoothness of the resulting graph (Fig. 1, *a*) cannot be interpreted, of course, as meaning that intensity and height are directly proportional, but only that the heights and the intensities progress together.

Titanium, scandium, and yttrium.—These related elements are interesting components of the solar atmosphere and may conveniently be considered together. A common characteristic is the high proportion of enhanced lines. As clear a separation as possible has been made between the normal or unenhanced lines and the enhanced lines. As there are several degrees of enhancement, it is probable that some enhanced lines still remain. For titanium, the table given by Reese[1] has been of service, and the information given by Mitchell's tables has rendered it possible to make a rather close separation for all elements. The large number of enhanced lines in the titanium spectrum shows so clearly the behavior of this class of lines that it has furnished criteria for separating the lines of the other elements.

TABLE III

TITANIUM, SCANDIUM, AND YTTRIUM

	Number of Lines								Element
	7	19	18	18	21	14	9	4	
Solar intensity	ooo	oo	1	0	2	3	4	5	Titanium
Flash intensity	0.1	0.25	0.4	0.9	1.5	1.9	2.8	2.8	Fig. 1, *c*
Height	271	290	347	353	389	429	494	487	

	Number of Lines						Element
	5	9	5	10	3	3	
Solar intensity	ooo	oo	0	1	2	3	Scandium and yttrium
Flash intensity	0	0.25	1.0	1.6	1.3	1.0	Weighted means
Height	290	295	360	385	483	433	Fig. 1, *e*

Lanthanum group = La, Ce, Dy, Er, Eu, Gd, Nd, Pr, Sa.—The elements La and Ce, according to the conclusions of Adams from his investigations on the spectrum of the limb and center of the sun and from his study of solar rotation, are at a low level in the solar

[1] *Astrophysical Journal*, **19**, 322, 1904.

atmosphere, and the same conclusion was reached in the writer's discussion of the data obtained from the displacements in the penumbrae of sun-spots.

Carbon.—It is probable that the lines referred to carbon are due to cyanogen[1] or to nitrogen, but for convenience of reference, the older designation will be used. Grotrian and Runge have recently obtained the cyanogen bands in the presence of nitrogen, when both cyanogen and carbon were evidently absent.[2]

The low level of carbon was remarked by Adams in his rotation work, and by the present writer in the second paper on radial motion where reference was made to the weakening of the carbon or cyanogen lines in the arc in the presence of metallic vapors.[3] The high proportion of the weakest carbon lines on the chromosphere plates of Hale and Adams and upon the eclipse plate of Mitchell is evidence of the low-level origin of these lines.

TABLE IV

LA GROUP AND CARBON

	Number of Lines							Element
	1	7	27	18	8	10	2	
Solar intensities	oooo	ooo	oo	o	1	2	3	La group
Flash intensities	0.0	0.7	0.9	1.1	1.8	0.8	2.0	Fig. 1, *g*
Heights	250	329	354	382	406	395	550	

	Number of Lines						Element
	6	13	10	7	7	4	
Solar intensities	oooo	ooo	oo	o	1	2	Carbon
Flash intensities	0.7	0.5	0.2	0.9	0.7	1.8	Fig. 1, *i*
Heights	342	342	335	443	479	738	

The curves showing the relation between height and the Rowland intensities for the elements Ti, Sc-Y, the La group, and CN are given under *c*, *e*, *g*, and *i* in Fig. 1. When one considers the number of observations upon which the points depend, the uncertainties that occur in estimating the lengths of arcs, and the

[1] Kayser, *Handbuch der Spectroscopie*, **5**, 229.

[2] *Physikalische Zeitschrift*, **15**, 545, 1914.

[3] *Mt. Wilson Contr.*, No. 74, pp. 6–7; *Astrophysical Journal*, **38**, 162–163, 1913.

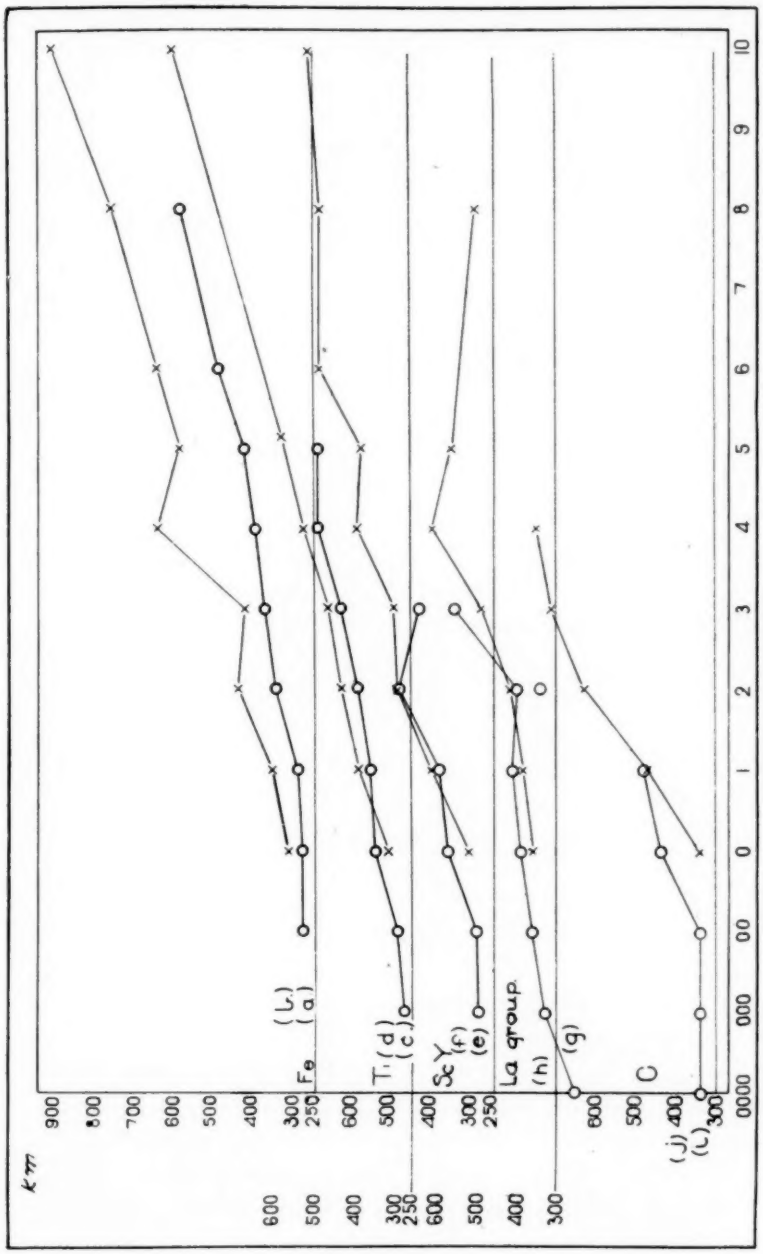

FIG. 1

O relation between solar intensity and height
× relation between flash intensity and height

increased regularity that accompanies increased weight, the connection between Rowland intensity and height is quite evident.

An estimation of the accuracy to be expected in the reduction of eclipse plates may be had from considering the heights and flash intensities. Mitchell says that "equality in height must entail an equal intensity in the flash spectrum."[1] In Fig. 1 are shown graphically the relations between heights and flash intensities, and though more observations are at our disposal, the regularity of the graphs is no greater than when Rowland intensities are used as abscissae. There are discordant observations in both cases, lines of equal flash intensity giving widely different heights, lines of very unequal flash intensities giving equal heights, and lines of the same solar intensity giving different heights—and this last is particularly true for elements with strong lines to the violet of λ 3850, in which region the apparatus was much less sensitive, owing to the low reflecting power of silver and speculum metal and to the decreased sensitiveness of the photographic plate. These discordant observations do not weaken the general statements that lines of equal mean intensity correspond to equal heights, and that lines of a given flash intensity mean equal heights, but rather tend to strengthen them, for if, in spite of these discordances, the means show that heights progress with both Rowland and flash intensities, it seems there must be a fundamental truth underneath the statements.

In saying that the Fraunhofer lines of intensity 4, for example, are at a definite elevation in the solar atmosphere, it is meant that when a sufficiently large mass of data is treated statistically it results that the lines of intensity 4, as a group, are of a higher level than those of an intensity less than 4, and at a lower level than groups of intensities greater than 4. The case is quite similar to that of stellar distances and proper motions. It cannot be said of a definite star with large proper motion that it is a near star, but out of a hundred stars with large proper motions it can be quite safely assumed that a very large majority are of the near-star group. There are undoubtedly differences between the effective levels of lines of equal solar intensity, just as the writer showed that there

[1] *Astrophysical Journal*, **39**, 128, 1914.

are differences in the radial displacements given by lines of like solar intensity, and that these differences are related to the classification of iron lines on other grounds. A thing that is generally true must first be established in its broad outline, and later, when some individual members of a group show themselves as definite exceptions, they offer a new means of obtaining added information and of pushing the analysis yet a step farther. It is yet too early to emphasize the exceptions, though data are accumulating that promise the added information. In the writer's opinion the criterion demanded by Mitchell in discussing this question is too exacting, when he requires that all lines of a given solar intensity must always be represented in the flash spectrum by lines of equal height.[1]

Some idea of the weight to be given to the means of the groups containing lines of like solar intensity may be had from an analysis of the group made up of solar intensity 3. This contains 72 lines and has a mean height of 369 km. Mitchell's estimates are made in 50 km steps so that heights of 350 and 400 km come nearest to this mean. A high degree of precision in any single measurement is an error of one unit of the scale, which in this instance is 50 km, so that heights of 300 and 450 km would still be within the range of error. Ninety per cent of the lines are within these limits. The one line below the lower limit is assigned a height much less than the average for lines of the same flash intensity, and the six above the upper limit are assigned heights greater than the average for lines of their flash intensities.

TABLE V

HEIGHTS AND RADIAL DISPLACEMENTS OF IRON LINES

Solar intensities....	∞	0	1	2	3	4	5	6	7–9	10–40
Heights.....	275	279	288	344	369	397	425	488	590	806
Displacements.....	+0.034	0.030	0.028	0.025	0.023	0.021	0.019	0.016	0.010	0.002

A comparison of the eclipse results for the iron lines with the radial displacements for lines of the same solar intensities is of interest, as the increasing displacements with decreased intensity were interpreted as consequences of greater depths.

[1] *Astrophysical Journal*, **39**, 128, 1914.

The progressive decrease of displacements with increase of heights is a phenomenon that appears to find its straightforward interpretation in differences of effective level; that is, at the lowest height given by the iron lines, the vapors are flowing out of the spot tangential to the surface with the highest velocity, at greater heights the velocity is less, and the lines of greater solar intensity originate at these higher levels.

THE DISTRIBUTION OF THE ELEMENTS

When one considers the maximum height at which the different elements can be detected by flash spectra, a vertical distribution is shown that confirms the distribution obtained from the consideration of the sign and magnitude of the radial displacements; that is, the highest level is given by the H and K lines of calcium, the next by the hydrogen lines, while the heavy elements appear only in the lower portions of the solar envelope.[1] The vertical section of the solar atmosphere, in Fig. 2, is drawn to scale from data taken from Mitchell's first paper, the mean values being used where there are several lines of the same solar intensity. Some related radial displacements are also given. As in the terrestrial atmosphere, the percentage composition of the heavy elements is greatest in the lower portions, the lighter elements gaining in percentage, but decreasing in absolute density with height.

The distribution of the elements in accordance with their atomic weights, and the march of heights with intensities indicated by eclipse data are in satisfactory agreement with the interpretation of radial displacements as phenomena of level. In the case of the heavy elements, lanthanum and cerium, the displacements in the penumbrae of spots are greater than for iron lines of equal solar intensities, and an interpretation on the basis of level places the origin of a line due to the heavy metal at a lower level than a line of the same solar intensity due to iron.

The flash spectra heights given by lines due to the heavy metals are greater than for iron lines of the same solar intensity, as Mitchell has shown in his second paper. In discussing the matter he says:

> It therefore seems that St. John and the writer both use the differences in heights or differences in level to explain differences in the lines of the solar

[1] *Mt. Wilson Contr.*, No. 74, pp. 3–8; *Astrophysical Journal*, **38**, 159–164, 1913.

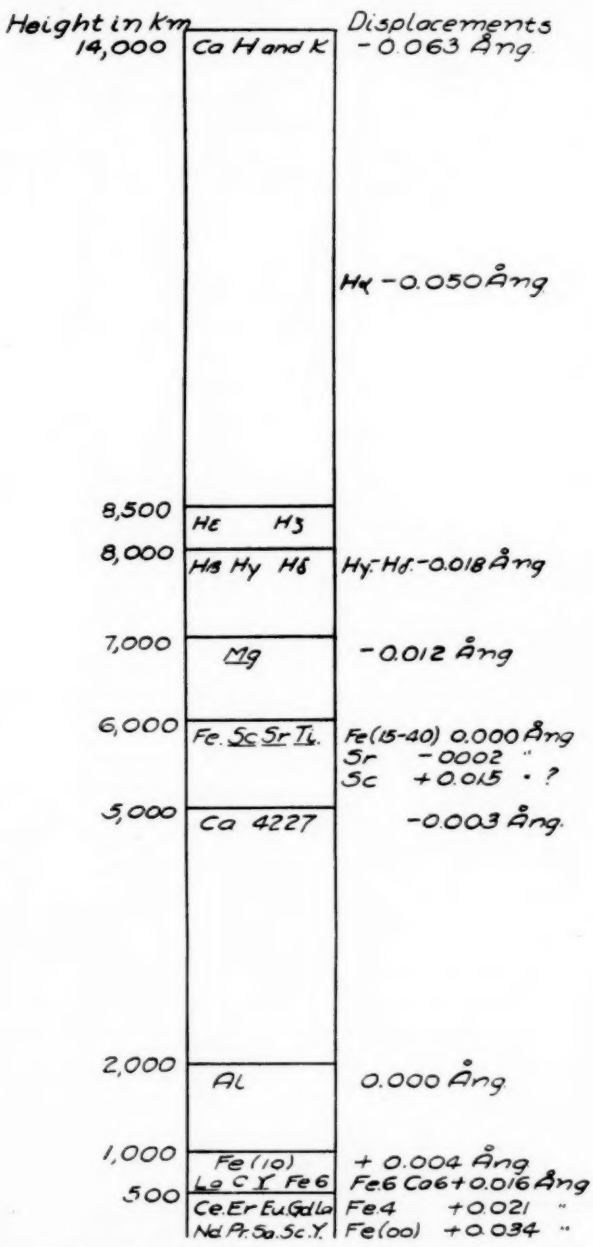

FIG. 2.—Absolute heights from flash lines; relative heights from displacements

spectrum. According to St. John's ideas, however, the rare earths are found in the low-lying regions of the reversing layer. Since they are found comparatively close to the photosphere, they are at a rather high temperature. As a result of this high temperature there is little absorption by the rare-earth vapors, and the Fraunhofer lines are not strong. These high temperatures, however, make more brilliant the lines of the flash spectrum, with the result that the intensities of the rare earths in the flash spectrum are much greater than in the Fraunhofer spectrum. Measures of the 1905 eclipse spectrum confirm the increased intensities in the flash spectrum demanded by St. John's theory, *but these measures do not show that the rare earths are found in shallow layers*, but exactly the reverse.[1]

The contradiction implied in the paragraph is only in appearance. Professor Mitchell informs the writer that he himself is not referring here to the absolute heights to which the rare-earth vapors extend, but to the fact that for lines of the same solar intensity the heights are greater in the case of the rare earths than for iron. My own conclusion was that, in the main, the heavy and rare-earth elements occur in detectable amounts only in the lower portions of the solar atmosphere, a conclusion that does not depend upon whether the lines of the heavy elements have their origin somewhat above or below the lines of iron of the same solar intensity, since none of the lines of the heavy elements reaches an elevation exceeding a few hundred kilometers, and with this conclusion Professor Mitchell writes that he is in entire agreement.

In Mitchell's second paper, the heavy elements (mean atomic weight, 148) are grouped with the light elements Sc (atomic weight, 44) and Y (atomic weight, 89) which are rich in enhanced lines, and it is possible that the heights reached by these elements have been mainly responsible for the high elevation of the group. In his Table III, p. 176, under solar intensity 3 to 5, there are 243 lines of the Fe group, with a mean height of 505 km, and 29 lines of the rare earths, with a mean height of 785 km. It is difficult to identify completely the 29 lines, but there are 22 lines attributed to Sc or Y only with a mean height of 864 km. There remain 7 lines with a mean height of 529 km to be accounted for. There are 4 lines of mean height 625 km, in which Sc or Y is the chief element, and 11 lines of mean height 491 km, in which one of the heavy-earth elements is the only or chief component. It is quite evident that the

[1] *Astrophysical Journal*, **39**, 128, 1914.

high elevation of 783 km is not due to the heavy elements. As to the higher elevation of the group when lines of solar intensity 6 to 10 are considered, it needs only to be said that no line of solar intensity 6 or more is known for the heavy elements, so that they have no influence here. The high elevation and flash intensity shown by the lines of the heavy elements relative to other lines of the same solar intensity appear prominently with decrease in solar intensity. The explanation seems to be found in the increased intensity of the flash lines of the heavy elements relative to the solar lines, which Mitchell says his measures confirm. An increased flash intensity due to the higher temperatures at the lower level carries with it an increase in height deduced from the length of the line. Hence it would seem that the heights given by flash spectra may be quite fictitious in the case of lines of very low level. The question is important in the interpretation of eclipse spectra and deserves further consideration.

FLASH SPECTRUM HEIGHTS FROM WEAK SOLAR LINES

The intensity assigned to a flash line is a complicated function of the length, width, and blackness of the line. If exposures of the same length could be given to the entire depth of the solar atmosphere and of sufficient duration to photograph the relatively cool H and K level, something like the following would be expected: a long, broad arc of weak intensity for the upper calcium, and short, narrow, and very black arcs for the lines of the La group, their blackness being due to the extremely high temperature of their vapors. For these lines the blackness would play an increasing rôle in the estimation of their intensities. A similar influence would make itself felt only in a less degree when determining the intensities on an ordinary eclipse plate; that is, the short arcs produced by the hot, low-lying vapors would be blacker in proportion to their length than the long arcs, and their intensities relative to that of the solar lines would increase with decrease of level. This increase of relative intensity with decrease of solar intensity, or increase of depth, is a marked feature of Mitchell's observations.

The exposure on the Mitchell eclipse plate was progressive, and hence the exposure at high levels was longer than at low levels. The highest level photographed is 14,000 km; the unenhanced

lines of the heavy elements are all below 500 km. Had the exposure begun at the instant the projected limb of the moon was 14,000 km above the photosphere, the exposure for the outer envelope would have been nearly 30 times as long as for the region where the unenhanced lines of the La group originate. The actual case differed from this only in degree. Had the same time of exposure been given for the lowest levels as for the highest, the lines of very weak solar intensity would have been still stronger, and had the exposure for all levels been as short as for the very lowest, it is a question whether the relatively cool high-lying vapors would have photographed at all. In the case of eclipse plates, a strong emission of the low-lying vapors is shown by the presence of the flash reversals of the weakest Fraunhofer lines, though the exposure time for them is practically instantaneous.

To obtain an idea of the general case, the mean flash intensity was found for all lines of solar intensity 6 or less assigned to a single element and the difference F−S taken as shown in Table VI. The regularity of the increase of flash intensity relative to the Rowland intensity is very marked from a flash strength of 3 intensity intervals less to a strength of 3 intervals more than the Rowland intensities in passing from solar intensity 6 to 0000. A low level in the solar atmosphere appears to be favorable to a high flash intensity relative to solar intensities of the corresponding lines, carrying with it increased length of arc, and this high relative strength is characteristic of carbon and the La group of heavy elements, in which cases it reaches its highest value.

TABLE VI

FLASH INTENSITY RELATIVE TO SOLAR INTENSITY

	Number of Lines									
	18	69	164	166	189	185	166	109	71	44
Solar intensity....	0000	000	00	0	1	2	3	4	5	6
Flash intensity ...	0.44	0.3	0.38	0.55	1.1	1.3	1.9	1.9	2.3	2.8
F−S	+3.44	+2.3	+1.38	+0.55	+0.1	−0.7	−1.1	−2.1	−2.7	−3.2

Another interesting fact appears when the heights for the same flash intensities of different groups of elements are compared with

the heights for the same flash intensities of iron. For 124 lines of the heavy elements and carbon, the heights average 76 km higher than for Fe lines of the same flash intensities; for 159 lines of Ti it is 5 km and for 73 lines of Sc and Y 2 km higher than for iron. It seems clear that some character of the flash lines of the heavy elements and carbon—the increased blackness, perhaps—has made it possible to follow the vanishing tips of the lines with less difficulty than in the case of the other elements and this has resulted in a greater length of arc for the flash lines of the heavy elements than for lines of the same flash intensity due to iron. When the heights for lines of the same solar intensities are compared, the average height for the heavy elements and C is 104 km, for Ti 54 km, and for Sc and Y 69 km greater than for the Fe lines. Since the flash lines of Ti, Sc, and Y yield the same heights as the flash lines of Fe of like intensity, the level of the lines of these elements relative to Fe lines of the same solar intensity is indicated with greater probability than for the heavy elements and carbon, in which case the high level relative to iron lines of like solar intensity is mainly accounted for by the greater height given by lines of the same flash intensity. An indication of this effect of low level appears in the graphs showing the relation between heights and solar intensities. For lines of medium intensity the slope of the curves is regular when the points depend upon a sufficient number of lines; but for lines of low intensity the heights decrease less rapidly or not at all. In the case of these lines, the actual differences in level are smaller than for stronger lines, and the increased emission due to the higher temperatures of the lower levels would tend to increase their blackness and to give, as shown above, a slightly anomalous height on eclipse plates, the effect becoming more apparent with decreasing intensity or increasing depth.

The relative intensity of the lines on a flash spectrum plate plainly depends upon the phase of the eclipse photographed, the exposure times, and the sensitiveness of the apparatus for different wave-lengths. Bearing upon this question the following examination was made of the probable sensitiveness of Professor Mitchell's apparatus: A Seed orthochromatic plate was exposed in a concave grating spectrograph of 1 m focus to sunlight reflected from a

freshly burnished silver mirror, a combination similar to that used by Mitchell. The exposure and development were such as to end the spectrum between λ 5800 and λ 5900. The opacity of the plate was measured by a Hartmann microphotometer and the results plotted against wave-length. The maximum sensitiveness is near λ 4500, and at λ 3300 it is about one-third of the maximum. The combined reflecting power of silver and speculum metal was found from the data of Hagen and Rubens;[1] at λ 3300 the reflecting power is 0.23 of what it is in the visual region, so that the apparatus was approximately one-thirteenth as sensitive at λ 3300 as in the visual region.

This would give lower heights for lines in the ultra-violet. In a certain sense it would be like giving a shorter exposure to the ultra-violet lines of the high-level vapors, as the photographic effect of the radiation of the relatively cool high-level layers would be more decreased than that from the intensely hot low-lying layers, and the relative intensities of the high- and the low-lying lines would be greatly changed.

Evidence that the relative flash intensities are greatly altered in the ultra-violet region and change in such a way as to increase the intensities of the weak solar lines relative to the stronger one is found by comparing the values of F−S in the case of very weak medium, and very strong solar, lines for the two regions.

TABLE VII

		F−S	
		Violet	Ultra-Violet
Intensity	ooo–o	+1.1	+ 1.1
Mean intensity	3	+1.52	− 1.82
Intensity	20	+8.4	−16.2

The intensity of the weak lines relative to the solar lines remains practically unchanged, but the relative intensity decreases progressively with increased solar intensity. The absolute flash intensity of ultra-violet lines of solar intensity 20 is a third of that for the violet lines of the same solar intensity, or the intensity of the

[1] Wood, *Physical Optics*, p. 466, 1911.

weak lines relative to that of the strong lines has increased threefold, entailing a great difference in their relative heights.

That the ultra-violet weak solar lines were photographed at all under these conditions is an indication of the enormous radiation from these low-lying layers, and the weakening of the flash lines which are due to the vapors extending to high elevations is what would be expected when the source of the flash line is a widely extended surface of relatively low intensity. In the one instance we have an intense emission concentrated in a small area; in the other, the source is a very extended surface of cooler vapor with a relatively feeble emission.

In view of the preceding considerations, the usual method, in discussing eclipse plates, of determining heights from length of arc appears of doubtful value in the case of low-level lines.

There are several facts established by observation, relative to the members of the group of heavy elements, all of which should be considered in an attempt to fix the relative level at which the lines of these vapors occur.

1. The intensity of the La and Ce lines at the sun's limb is greatly decreased in comparison with the intensity at the center, while that of most lines is increased. (Adams.)

2. The rotation value given by La and carbon is lower than for Fe lines of the same solar intensity. (Adams.)

3. The displacements given by the La and Ce lines and those of the rare-earth metals in the penumbrae of sun-spots are very large, larger than for the Fe lines of the same solar intensity. (St. John.)

4. The heights given by Mitchell are greater for the flash lines of the La group and carbon than for the Fe lines of the same flash intensity.

5. The heights found by Mitchell are greater for the lines of the La group and carbon than for iron lines of like solar intensities.

Mr. Adams has shown that the behavior of the La and Ce lines at the sun's limb finds an explanation in their low level, that the angular velocity of the solar vapors decreases from that of the high-level hydrogen to a minimum value for lanthanum and carbon, and that the low rotation value for these lines relative to that for

Fe lines of the same solar intensity finds its obvious cause in the low level of these lines relative to the Fe lines of the same solar intensity. The radial displacements of the Fraunhofer lines in the outer edge of the penumbrae of spots appear to be a Doppler effect and the regular increase with decrease of solar intensity, a phenomenon of level. Under this interpretation, the greater displacements given by the lines of the heavy elements than by Fe lines of the same solar intensity mean obviously a low level for these lines relative to those of Fe of like solar intensity.

The greater heights given by the flash spectrum for the lines of the heavy elements relative to those of iron of like solar intensity appear to Mitchell to be in contradiction to these three lines of observation. A large part of this excess in height comes, however, from the character of their flash lines, which has allowed their vanishing tips to be followed farther than in the case of the lines of iron of the same flash intensity. This characteristic of their flash lines, rather than their greater strength relative to the Fe lines of the same solar intensity, contributes probably the major part of the excess in height.

HEIGHT AND WAVE-LENGTH

In the discussion of the radial displacements, it was noticed that the displacements were greater in the red than in the violet for lines of the same solar intensity. This was attributed to our seeing into the sun to a greater depth in the red, owing to the greater scattering of the violet light—that is, the effective levels are lower for the longer wave-lengths. Inasmuch as it is the upper border of the effective layer, or the height to which the vapor emitting a given line extends, that determines the length of the arc, a similar effect might be expected from the flash spectrum. A large number of lines—666—of solar intensity 1 to 6, identified with single elements, were divided into groups of 100 A each. The mean heights for the groups were plotted as ordinates. The result is a curve, (Fig. 3, *a*), with a maximum at λ 3850, a slow drop to the red, and a rapid fall to the ultra-violet.

The maximum height given by the flash lines is at λ 3850, while the photographic maximum of the apparatus is at λ 4500. The

curve, Fig. 3, *b*, represents approximately the effective sensitiveness of the apparatus for various wave-lengths as it is derived from the photographic sensitiveness and the reflecting powers of the surfaces.

It will be noticed that the heights do not follow the effective sensitiveness, that in the region λ 3700 to λ 4350 the heights are greater than in the region λ 4350 to λ 5000, though the sensitiveness is greater in the latter case, and that in the region λ 5000 to λ 5800 the heights are not low because of low sensitiveness of the apparatus, which for this region averages slightly higher than at λ 3850. Apparently the reported intensities and heights are fully as great as photographed, for Mitchell says that in estimating intensities allowance was partially made for the decrease in the sensitiveness of the plate in the green and yellow regions. With due allowance for plate sensitiveness and reflecting power of the surfaces, the fact still remains that the heights decrease with increasing wave-lengths.

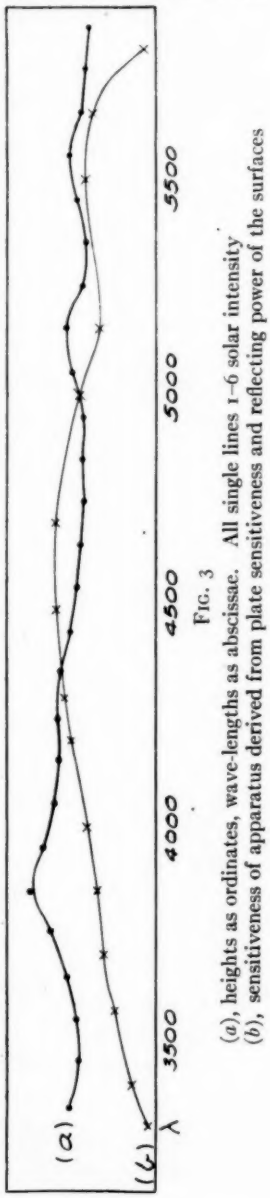

FIG. 3

(*a*), heights as ordinates, wave-lengths as abscissae. All single lines 1–6 solar intensity
(*b*), sensitiveness of apparatus derived from plate sensitiveness and reflecting power of the surfaces

EFFECTIVE LEVELS

The effective level of a line may be defined as that portion of the entire depth of a vapor that is mainly concerned in the production of the line. The light in a Fraunhofer line of intensity 4, for example, comes from a lower depth than in a line of intensity 10 because of the greater selective absorption in the latter. The light from the photosphere and from the lower, hotter layers of the vapor under consideration is selectively absorbed in

both instances and fails to reach the surface, so that the emitted light of the respective wave-lengths comes from more or less sharply bounded shells of vapor having a larger radius for the line of intensity 10 than for the line of intensity 4.

The idea of effective levels has been tacitly employed, though not so definitely expressed in all spectroheliograph work. There is no question among solar physicists but that the light employed in H_3–K_3 photographs comes only from the highest level when a very narrow slit is used; that light forming the H_2–K_2 photographs comes from a low level, but not from the lowest levels; these are successively reached by setting the slit on the broad H_1–K_1 shading at increasing distances from the center of the line. It is clear that here we are dealing with successive shells of calcium vapor. It is probable that a similar succession of shells would obtain for the calcium lines of decreasing intensities, and that is what appears in the radial displacements and in the heights from the flash spectrum when a sufficient number of lines is present to obtain means of weight. For the calcium lines the succession is indicated as follows:

	H_3–K_3	H_2–K_2	4227	8–6	5–3	2–1
Radial displ.	−0.063	−0.044	−0.003	+0.017	+0.019	+0.028 A
Flash sp. hgt.	14,000		5,000	583	491	350 km

The negative signs indicate an inflow, the positive signs an outflow, from the spots, tangential to the solar surface.

What may be called the radiation centers of gravity of the effective layers are at not greatly different levels in the reversing layer. Lines of solar intensity 0000 to 10 occur below 1000 km, allowing an average distance of about 70 km for each layer. If such be the conditions in the solar atmosphere, it is clear that flash spectra measurements of heights made by 50 km steps could show the facts only by taking the means of a very large number of lines, that individual departures from the mean would be numerous, and that small differences between the short arcs would be difficult of determination. In point of view of precision there is a wide difference between the estimates of flash intensities and arc lengths upon a single flash spectrum plate and differential measurements of line displacements upon many plates of high dispersion.

GENERAL CONCLUSIONS

The facts determined by observations that bear upon the question of the distribution of the elements in the solar atmosphere are gradually increasing in number and are already sufficient for the deduction of some probable conclusions. The salient facts shown by flash spectra and displacements in the penumbrae of sun-spots, based upon mean values, are as follows:

From Flash Spectra	From Radial Displacement
The heights and flash intensities increase progressively with the solar intensity of the lines.	The displacements vary progressively with the solar intensity, from large negative values for the strongest lines to large positive values for the weakest lines.
The flash intensity of unenhanced lines relative to the solar intensity increases with decreased solar intensity as follows in the case of iron:	The displacements increase with decrease of solar intensity as follows in the case of iron:

Solar int.	oo	o	1	2	3	4	5	6	7–8–9	10–40
F–S	+1.25	+0.26	−0.67	−1.2	−1.6	−2.4	−3.0	−2.5	−3.9	−9.4
Displ.	0.034	0.030	0.028	0.025	0.023	0.021	0.019	0.016	0.011	0.002

The vapors of the elements with the strongest solar lines extend to the greatest heights.	The elements with the strongest solar lines give small positive or negative displacements.
Enhanced lines are higher than the normal or unenhanced lines of the same solar intensity.	The displacements are smaller for enhanced lines than for normal or unenhanced lines of the same solar intensity.
The heights are greater for lines in the violet than in the yellow-green when lines of the same solar intensity are compared.	The displacements are less for lines in the violet than in the yellow-red when lines of the same solar intensity are compared.

These facts are all harmonized by the consideration that the vapors of the elements ascend in detectable amounts to different heights, that the lines of any one element originate at depths increasing with decrease of solar intensity, that the enhanced lines are

higher than unenhanced lines of equal solar intensity, and that we see into the sun to greater depth at the red end of the spectrum than at the violet.

The general distribution of the elements in the solar atmosphere and the relative levels or heights for the lines of a given element, such as iron, were found by interpreting radial displacement in spots as Doppler effects varying with the depth. This interpretation finds strong confirmation in Mitchell's eclipse data, which give a similar distribution of the vapors of the elements and show that the heights of the lines of an element rich in lines, such as iron, increase progressively with their solar intensities. Thus we have added evidence that in the Evershed effect we have a means of sounding the solar atmosphere with a high degree of precision.

The writer finds no better expression for summing up this investigation than that used at the conclusion of the discussion of the displacements in the penumbrae of sun-spots, excluding the specific reference to the H_α line which does not appear in Mitchell's flash spectrum: "The resulting distribution shows that the H_3 and K_3 lines of calcium are the lines of highest level, followed by the H_α line of hydrogen, and that, in the main, the heavy and rare elements occur in detectable amounts only in the lower portions of the solar atmosphere."

MOUNT WILSON SOLAR OBSERVATORY
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NEW "VAPOR LAMPS" AND SOME PRELIMINARY OBSERVATIONS OF THEIR SPECTRA IN THE SCHUMANN REGION

By FREDERICK A. SAUNDERS

It is now many years since mercury arc lamps became familiar to physicists. Similar lamps working with the vapors of cadmium or of zinc have more recently been made.[1] Five years ago, Professor Paschen[2] succeeded in running a lamp with the vapor of thallium. At his suggestion, and with his active personal cooperation, I undertook to make a calcium vapor lamp. The work was done in Professor Paschen's laboratory, and my very best thanks are due to him, not only for the design of the vapor lamp and for his skilful help in innumerable experimental difficulties, but most of all for the extraordinary generosity and kindness with which he placed the entire resources of his wonderful laboratory at my disposal. In the end, vapor lamps were successfully run in a quartz tube both with calcium and with magnesium. The spectra of these lamps were examined from λ 9000 to λ 2000, and, in a preliminary sort of way, in the Schumann region. The first part of this work will be reported on later; the present paper concerns itself with (1) the new vapor lamps, and (2) the experiments in the Schumann region.

THE NEW VAPOR LAMPS

A quartz tube was secured of the shape shown in the figure. Its dimensions were, roughly: length, 30 cm; internal diameter, 1 cm; distance between terminals, 7 cm. The ends were ordinarily closed by plates of quartz, Q_1 and Q_2, attached by white sealing-wax and kept cool by wet cotton. The terminal-tubes at A and B were each filled with about 1 cc of small lumps of calcium (or magnesium) metal, into the under part of which were inserted copper wires. These wires were led out through long, narrow, bent

[1] Stark and Küch, *Physikalische Zeitschrift*, **6**, 438, 1905.

[2] *Annalen der Physik*, **29**, 628, 1909.

tubes, dipping under water (for cooling) and sealed air-tight at the ends by wax. Connection with an air-pump was established through a third tube. A low pressure was easily produced by a pair of pumps in tandem, driven by a motor—one, a rotating oil pump, the other, a Gaede mercury pump. The terminal wires were attached to 220-volt d.-c. mains, and the secondary of a small induction coil was connected between the negative terminal of the lamp and an idle terminal in the air-pump connections (an external electrode is sufficient).

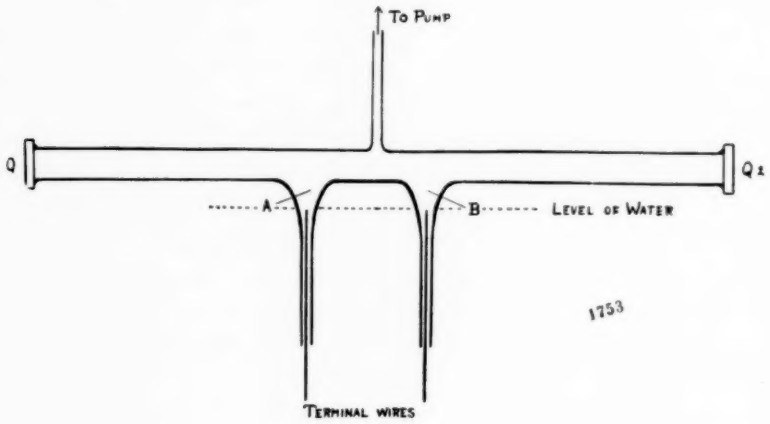

FIG. 1

With a blast lamp the middle portions of the tube and the metal in the electrodes were heated red hot and exhausted until green fluorescence showed itself in the glass connections of the apparatus when the induction coil was operated. After this process was thoroughly carried out, the arc usually started easily; sometimes it was sufficient merely to close the 220-volt connection, and ionize for a moment the residual gas in the tube by a discharge from the induction coil; but more often a certain amount of preliminary heating was also necessary.

The arc itself was intensely brilliant; in the case of calcium, of a bluish-white color, in magnesium greenish-white. Almost immediately after the arc began, the metal covered the inside of the tube with an opaque deposit, extending, however, over the

central portion of the tube only. The quartz windows at the end, through which the observations were made, remained clear for a long time if the tube was air-tight and the current not more than 4 or 5 amperes; in one case, they were still clear after a three-hour run. A larger current, or a leak, extended the deposit of metal to the windows also. At the instant of fusion a good deal of gas was usually liberated, which was pumped away, but the pump could then be disconnected.

In the case of magnesium, the melted metal did not at first flow down about the copper terminal wire; the latter fused immediately and broke the circuit. It was necessary to melt about the wire a small quantity of metallic tin beforehand, in order to maintain contact; by this device the arc was made continuous. This introduced no difficulties, and the resulting spectra were free from the lines of tin.

One could not help admiring a material such as quartz which could tranquilly endure an ordeal of the sort that it was here put through. The terminal tubes dipped into water. The temperature inside them was probably extremely high; and, outside, within 2 or 3 mm of the water, one of the terminals was usually red hot. The tube proved to be fairly durable, and lasted for nearly a dozen runs with different metals. It was, however, found to be roughened internally after every long run, and the life of the tube was prolonged by re-fusion of the roughened parts each time in an oxy-hydrogen flame. The tube walls were also thickened where the action was greatest.

The column of luminous vapor obtained with this tube probably averaged at least 6 cm in length, and the spectra obtained by end-on observations throughout this column are, as one might expect, remarkable for the relative strength of faint lines, especially the outer members of the series. All lines are sharp, and the spectra are almost free from bands.

A noteworthy property of the calcium lamp (and probably with magnesium also) is its ability as an air-pump. The vapor absorbs gases with which it can react chemically. Oxygen and nitrogen are both quickly removed, and hydrogen apparently also to some extent. Argon is not affected. The result is that,

if there is a small leak in the connections, the lamp becomes a little reddish in color and numerous argon lines appear in the spectrum. An arc spectrum of argon is perhaps a novelty but it proved indistinguishable from the usual "red" discharge-tube spectrum.

EXPERIMENTS IN THE SCHUMANN REGION

It was noted in connection with the new lamps that the deposit of metal did not extend more than a few centimeters beyond the column of luminous vapor. It seemed to me likely, therefore, that any light belonging to the Schumann region would suffer very little absorption in passing from the source to the (nearly) empty space at the ends of the tube. Accordingly, the lamp was attached directly to a vacuum grating spectrograph, one of the quartz plates having been removed from the end of the lamp for this purpose. The light thus had no "window" to pass through, but went directly from the luminous vapor through the slit to the concave grating, and was returned to the Schumann plate, passing on its way through nothing but the *very* small amount of residual gas (mostly hydrogen) in the spectrograph. Lyman[1] has used a somewhat similar arrangement with hydrogen, but these experiments differ from his in that the vacuum in the spectrograph was higher, and the method is applicable to a number of metals. By its use, it should be possible to study several spectra to a limit considerably beyond what has already been attained. The experiments here described had to be discontinued almost at the beginning, but they went far enough to show that the method is a good one. The spectra of zinc and of calcium, and incidentally of hydrogen, were photographed to a limit somewhat beyond λ 1000.

The vacuum spectrograph was similar in general plan to that devised by Lyman. It consisted of a 1-meter concave grating (purchased through the generosity of the Rumford Fund Committee, for which I wish here to make grateful acknowledgment) mounted at a slight angle to the normal inside one end of a brass tube. Both ends of this tube were closed by brass caps, fitting

[1] *Astrophysical Journal*, **19**, 203, 1904.

the tube by ground conical joints, and readily removable. The cap on the end away from the grating carried a slit, outside of which sources of light could be sealed-on, and at one side of the slit was the photographic plate. The latter could be changed without disturbing the source, or the slit, by removing a flat plate, with ground surfaces, which formed the back of the cap.

A portion of the slit was covered with a small quartz plate, transparent to about λ 1600. If any ghosts of the type discovered by Lyman had been present in the region of still shorter wavelength, they would have been generated by strong lines of wavelength greater than this limit and would have been transmitted by the quartz. Since the grating gives practically no astigmatism in the first order with this arrangement, the false lines would have been longer than the real ones. None was found, and the method enables one to make sure that the new lines are all real.

The evacuation of the tube had to be done afresh for each plate inserted in the spectrograph, and the volume to be emptied was about 7 liters. There was a small discharge-tube connected to the spectrograph to indicate the state of the vacuum attained. With the pumps mentioned above, when everything was tight (not always the case!) green fluorescence appeared in the discharge-tube in less than three-quarters of an hour. The apparatus was washed out, generally more than three times, with hydrogen, each time exhausting the main tube to a pressure of less than a millimeter, and then letting in about 15 cc of hydrogen. After three washings, the spectrum of the discharge-tube showed nothing but hydrogen, with possibly a trace of mercury vapor from the pump. The entire operation of evacuating to green fluorescence in preparation for a run with the calcium lamp could be accomplished in about an hour. The self-evacuating power of the lamp maintained the vacuum afterward, and even carried it still higher. On one occasion, after an hour's run, no discharge would pass through the discharge-tube at all. This meant that the calcium lamp had exhausted the 7-liter spectrograph through a slit 5 mm by .04 mm to a much lower pressure than it started with, and this was accomplished in spite of the probable emission of gas from the walls of the brass tube during this time.

THE HYDROGEN SPECTRUM

Lyman has very recently[1] extended our knowledge of this spectrum to λ 905, and has found that the Ritz series behaves as though it were independent of the secondary spectrum. His results are here confirmed.

The preliminary discharge necessary to start the calcium lamp gives the secondary spectrum of hydrogen exactly as Lyman found it. This spectrum was also photographed by itself, for comparison. On all the plates taken with the lamps, a few of the strongest lines of this spectrum are faintly visible, probably on account of the preliminary discharge used to start the arc. On every vapor-lamp plate, however, with both calcium and zinc lamps, the hydrogen line λ 1216 is present, and always very strong relative to the lines of the secondary spectrum. This is the first line of the Ritz series; the second line λ 1026 is also to be seen on two plates, one of which is practically free from the secondary spectrum.

We have here, therefore, further evidence that the Ritz series lines are independent of the secondary spectrum, and behave toward it in the same way as does the ordinary series in the visible spectrum.

THE CALCIUM SPECTRUM

The spectrum of the calcium lamp in the Schumann region is almost the same as was obtained by Lyman[2] from a spark. The narrow pairs are the most prominent feature; other faint lines were obtained, as shown in Table I, and there are probably still more to be found, as the Schumann plates used were not very good; but the identification of new lines is not easy, and cannot be carried out until we know the spectra of several more elements in this region. There seem to be no strong calcium lines between λ 1300 and λ 900. None are to be expected, excepting the later ones of the principal series of pairs (the series to which H and K belong); of this series, one line may be in the list but there is no way of making sure of it. The shortest observed line was at λ 977.9,

[1] *Nature*, May 7, 1914.

[2] *Astrophysical Journal*, **35**, 341, 1912.

possibly due to Si, though its origin must be left an open question. There were indications of lines still farther out, but too uncertain to be worth publishing.

TABLE I

SPECTRUM OF THE CALCIUM LAMP

Intensities	λ Vacuum (Saunders)	λ Vacuum (Lyman)	Remarks
1	1850.9	1851.3	
0	——	1843.8	
6	Standard	1840.2	
5	——	1838.0	
1	——	1828.1	Mg
2	——	1815.0	
1	——	1807.8	
4	Standard	1555.1	
3	——	1553.5	
1	1533.8	1533.4	
2	1475.5	.....	Probably Ca
2	Standard	1434.3	
1	——	1433.1	
1	1400.7	.....	Probably Ca
0	——	1370.6	
0	——	1369.1	
1	1347.1	.....	Probably Ca
0	1264.7	1264.5	? H
0	977.9	.....	Probably not Ca

Table I shows, in the first column, the intensities estimated on a scale from 10 to 0; in the second column, the wave-lengths measured from certain lines assumed as standards; a dash indicates a line observed but not measured. In the third column are the wave-lengths given by Lyman for all the lines observed here; his list includes others in addition. There are some faint lines on the plates which are not included in the table, as the evidence is decidedly against their being due to calcium. The accuracy of my measurements should be about the same as Lyman's (i.e., within 0.3 A) in the region down to λ 1200, but the error may well be somewhat larger beyond.

THE ZINC SPECTRUM

In the attempt to push the limit of the spectrum farther out, the lamp was run with calcium in the negative terminal and zinc in the positive one. The spectrum obtained was that of zinc only,

but was underexposed. It shows, however, a few of the zinc lines found by Wolff,[1] the two hydrogen lines λ 1216 and λ 1026, a few belonging to the secondary hydrogen spectrum, and a few others, possibly zinc, the shortest of which is at λ 1099.6. Apparently, zinc gives no very strong lines far out in the spectrum, but a longer exposure would be interesting to try.

TABLE II

SPECTRUM OF THE ZINC LAMP

Intensities	λ Vacuum (Saunders)	λ Vacuum (Wolff)	Remarks
4	Standard	1589.7	
3	Standard	1457.6	
1	Standard	1404.2	
0	1394.1	.....	? H
0	1364.0	.....	Probably H
0	1345.5	.....	Probably H
0	1342.6	.....	Probably H
0	1334.1	.....	Probably H
0	1104.2	.....	? Zn
0	1102.6	.....	? Zn
0	1099.6	.....	? Zn

SUMMARY

1. Arc vapor lamps, of the sort used by Paschen with thallium, have been successfully run with Ca and Mg in red-hot quartz tubes at a high vacuum, and their spectra have been obtained over a wide range.

2. The spectra of the Ca and Zn lamps have been examined in the Schumann region without passing the light through any fluorite, or other absorbing material, and lines, probably metallic, have been obtained as far out as λ 977.9.

3. The Ritz series of hydrogen has been photographed as far out as λ 1026 in the spectra of these lamps, and it is practically independent of the secondary spectrum.

4. The Ca lamp acts like a chemical air-pump, removing the gases with which the calcium combines.

TÜBINGEN
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[1] *Annalen der Physik*, **42**, 825, 1913.

SOME SPECTRAL CRITERIA FOR THE DETERMINATION OF ABSOLUTE STELLAR MAGNITUDES[1]

By WALTER S. ADAMS and ARNOLD KOHLSCHÜTTER

In the course of a study of the spectral classification of stars whose spectra have been photographed for radial velocity determinations some interesting peculiarities have been observed. The stars investigated are of two kinds: first, those of large proper motion with measured parallaxes; second, those of very small proper motion, and hence, in general, of great distance. The apparent magnitudes of the large proper motion, or nearer stars, are somewhat less on the average than those of the small proper motion stars, so that the difference in absolute magnitude must be very great between the two groups. The spectral types range from A to M.

The principal differences in the spectra of the two groups of stars are:

1. The continuous spectrum of the small proper motion stars is relatively fainter in the violet as compared with the red than is the spectrum of the large proper motion stars. The magnitude of this effect appears to depend on the spectral type, and increases with advancing type between Fo and Ko.

2. The hydrogen lines are abnormally strong in a considerable number of the small proper motion stars. Thus six stars which show the well developed titanium oxide bands characteristic of type M have hydrogen lines which would place them in types G4 to G6, and many others which show the bands strongly would be classified under type K from their hydrogen lines. That the spectra of these stars are not composite is shown by their radial velocities. The hydrogen lines in the spectra of the large proper motion stars which show the titanium oxide bands are without exception very weak.

3. Certain other spectrum lines are weak in the large proper motion stars, and strong in the small proper motion stars, and

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 89.

conversely. It is with the possibility of applying this fact to the determination of absolute magnitudes that the results given in this communication mainly have to deal.

I. INTENSITY OF THE CONTINUOUS SPECTRUM

A comparison of the intensity of the continuous spectrum of several pairs of stars of small and of large proper motion photographed upon the same plate was made recently by one of us, and showed a marked weakening relatively in the violet region for a majority of the small proper motion stars.[1] With a view to supplementing these observations with the larger amount of material available in the radial velocity photographs we have calculated the densities at several points in the spectrum for a considerable number of these stars, and compared the resulting values for the stars of small with those of large proper motion.

The method employed, though by no means so accurate as a photometric measure of density, appeared to be as good as the character of the material would warrant. It is evident, of course, that in the case of an individual star the conditions of transparency under which the photograph was taken, the zenith distance, and some other factors might influence the result seriously. For the mean of a considerable number of photographs and stars, however, it would seem that these effects should counteract each other largely, or at least be similar for the two groups of stars under comparison. No photographs have been included which were taken at great zenith distances.

The plan adopted for the determination of the densities was as follows: A standard plate of α Tauri was first obtained, several spectra taken with different exposure times being placed side by side on the negative. The photograph of each star was then compared with this standard plate under a Hartmann spectrocomparator, and estimates were made of the intensity of the continuous spectrum relative to that of α Tauri at three selected points at the violet and four points at the red end of the spectrum. The points were selected in regions as free from lines as possible. The estimates were made in tenths of a unit between the α Tauri spectra. Thus

[1] *Mt. Wilson Contr.*, No. 78, *Astrophysical Journal*, **39**, 89, 1913.

1.5 indicates an intensity half-way between the first and second of the standard spectra. After the comparisons had been finished the α Tauri photograph was measured under a microphotometer, and the densities were calculated at the points of comparison. The results for all of the stars were then reduced to densities.

The values for the groups of stars are given in Table I. The densities for the three violet wave-lengths have been combined to form a mean at λ 4220, and similarly for the four wave-lengths near λ 4955.

TABLE I

	Number of Stars	Average μ	Average Type	Density at λ 4220	Density at λ 4955
A0–A9	15	0".020	A2	0.30	0.32
	16	.13	A3	.29	.32
F0–F9	10	.012	F4	.25	.37
	23	.66	F6	.32	.37
G0–G4	8	.009	G3	.22	.41
	30	.64	G2	.33	.41
G5–G9	14	.011	G7	.25	.48
	22	.64	G7	.40	.48
K0–K4	24	.011	K2	.16	.44
	22	0.70	K1	0.31	0.44

The features of note in these results are:

a) The small proper motion stars of types F to K are decidedly weaker in the violet part of the spectrum than the large proper motion stars.

b) The difference is inappreciable for two groups of A-type stars for which the ratio of proper motions is 1:6.5.

c) The difference increases with advancing type from F to K, being twice as great for the latter. The ratio of proper motions for the groups of small and of large proper motion stars is nearly the same for the stars between F and K. Hence if interpreted in terms of distance the ratio of distances should be nearly the same, and it would appear that at least a part of the absorption in the violet part of the spectrum of the distant stars must be ascribed, not to scattering of light in space, but to conditions in the stellar atmospheres. In the case of the A-type stars the results are inconclusive, since the ratio of the proper motions shows that the negative result found may be due to the fact that the difference

of distance between the two groups of stars is insufficient to produce a measurable amount of scattering.

II. THE HYDROGEN LINES

The abnormal strength of the hydrogen lines in the spectra of certain of the small proper motion stars is of peculiar interest because of the possibility of selective absorption by hydrogen gas in interstellar space. The radial velocity affords a means of determining the origin of the additional absorption since it is highly improbable that the hydrogen in space would have the motion of the stars observed. Accordingly we have given especial attention to the determination of the radial velocities of these stars from the hydrogen lines as compared with other selected lines in the spectrum. The results obtained indicate that within the limits of error of measurement the hydrogen lines give essentially the same values as the other lines, and no differences have been found of an order to correspond to the abnormal intensity of the lines.

TABLE II

Star	Magnitude	Type	Metallic–H Lines	Computed Metallic–H Lines for Absorption	
				In Star	In Space
			km	km	km
Boss 539	5.6	G7p	−0.5	0.0	+13.4
707	5.6	G9p	+0.3	.0	+ 4.3
1309	5.8	G9p	+0.6	.0	−16.7
1560	5.3	G5p	+1.6	.0	+15.3
1846	5.7	G9p	+2.9	.0	−50.9
2020	5.8	G6p	+0.3	.0	−27.7
2144	6.2	G6p	+4.9	.0	+15.6
2378	6.3	G6p	+2.2	.0	+ 6.4
2612	5.6	G8p	−2.0	.0	+ 1.2
2915	6.0	G5p	+0.7	.0	−14.9
3867	6.0	G6p	−1.2	.0	−18.9
4159	5.6	G5p	−3.6	.0	−17.2
5125	5.6	G5p	+1.8	.0	+ 3.4
6089	5.4	G4p	−2.2	.0	−31.7
6145	6.0	Fop	−4.6	0.0	0.0

In Table II are collected the results for 15 stars which show abnormal strength of the hydrogen lines most prominently. All of the stars except Boss 6145 have the bands characteristic of

type M. The classification given is based on the hydrogen lines. The column designated "Metallic—H Lines" gives the values in kilometers of the differences in the velocities derived from about 12 selected metallic lines and those from H_γ and H_β; a small systematic correction is applied to the latter, due probably to the effect of blended lines. These differences would, of course, be zero if all of the hydrogen absorption occurred in the stellar atmospheres. If it all occurred in space the differences would be those given in the final column on the assumption that the absorbing gas is at rest in space. The quantities are derived by applying to the velocities of the stars obtained from the metallic lines the corrections to these velocities for the motion of the sun in space.

If any appreciable hydrogen absorption occurred in space the differences, Metallic—H Lines, should, of course, be intermediate between the quantities in the last two columns. When, however, we combine the values for all of the stars, assigning weights according to the numbers in the last column, we find that 98 per cent of the hydrogen absorption must occur in the stellar atmospheres, and that but 2 per cent can possibly be due to hydrogen gas in space. This amount is far below the limits of accuracy of the observations.

III. THE RELATION OF LINE INTENSITY TO ABSOLUTE MAGNITUDE

Systematic differences of intensity for certain lines between stars of large and stars of small proper motion soon became evident in the course of the study of the spectral classification of these stars. In order to secure an accurate system of classification as well as to investigate these differences the following method was adopted. Pairs of lines were selected not far from one another in the spectrum and of as nearly as possible the same intensity and character, and estimations were made of their relative intensities. For classification purposes a line decreasing in intensity with advancing type, such as a hydrogen line, was combined with a line increasing in intensity with advancing type, such as an ordinary metallic line. In addition to these pairs used for classification purposes several pairs were selected which included all lines suspected of systematic deviations in certain stars.

The estimations were made on an arbitrary scale extending from 1 to about 12, 1 being the smallest difference in intensity which could be detected. The method, therefore, is analogous to the *Stufenmethode* of Argelander used in estimations of variable stars; hence, for physiological reasons, our scale will be approximately proportional to the logarithm of the intensity differences of the two lines. In general three plates were used for each star, and the photographs of the large and the small proper motion stars were intermingled in order that systematic effects on the estimation scale might be avoided.

After all of the estimations had been completed the material was reduced uniformly, and the results were examined with two objects in view: first, to investigate the changes of the estimated intensity differences with the spectral type, and on this basis to form a classification depending on certain well defined criteria; second, after correcting for changes with type to investigate changes with absolute magnitude.

An examination of the pairs of lines used for estimation indicated that the following pairs showed the largest and most definite changes with type. The Harvard scale of classification has been followed closely.

F8–G6 stars: $\frac{4227}{H_\gamma}$; $\frac{4326}{H_\gamma}$; $\frac{H_\gamma}{4352}$; $\frac{H_\gamma}{4405}$; $\frac{H_\gamma}{4384}$

G6–K9 stars: $\frac{4326}{H_\gamma}$ (wt. $\frac{1}{2}$); $\frac{H_\gamma}{4352}$; $\frac{H_\gamma}{4405}$ (wt. $\frac{1}{2}$); $\frac{H_\beta}{4872}$; $\frac{H_\beta}{4958}$

These lines, accordingly, have been used to determine the type of each individual star, and since no systematic differences for the different lines have been found, the mean of the determinations from the five pairs has been used as the final result for the spectral type. This method of classification has proved most satisfactory in use, and shows good internal agreement. The mean error of one determination depending on three plates is +0.4 subdivision of the Harvard scale, equal, for example, to the interval from G5.0 to G5.4.

As soon as the spectral type of each star had been obtained in this way, the results for the remaining pairs of lines were examined

with a view to seeing whether all of them fell into agreement with the classification, or whether there were systematic differences for different groups of stars. For this purpose we constructed a normal curve for each pair of lines from the stars of rather low absolute luminosity, plotting as abscissae the spectral types, and as ordinates the estimations of intensity differences. Finally we formed for all of the stars the differences between our estimations of relative intensity and the values from the normal curve corresponding to the spectral type. These differences, combined into means for two separate groups, are shown in Table III.

At the head of each column of ratios is given the mean of the absolute magnitudes of the stars observed. Thus for the F8–G6 stars the mean of the absolute magnitudes of the small proper motion stars is -2.9, of the large proper motion stars, $+6.1$. Although the number of stars used in the estimate of the ratios of the different pairs of lines varies somewhat, the same mean magnitude, which was derived from all of the stars, is used throughout. The computation of the absolute magnitudes of the individual stars was made from the measured parallaxes where these were available. In the absence of such determinations, or when the parallax was very small or negative, the absolute magnitude was computed from the proper motion by aid of the parallax derived from the following formula:

$$\log \pi = -1.00 - 0.005m + 0.86 \log \mu$$

where m is the apparent magnitude and μ the proper motion. This formula is contained in an unpublished investigation by Kapteyn and Kohlschütter on the luminosity-curve of the K-type stars, and is based upon a discussion of the relation between proper motion and parallax for the K stars. The unit employed in the determination of absolute magnitudes is $0''.1$; that is, the absolute magnitude is the apparent magnitude of a star at a distance corresponding to a parallax of $0''.1$.

The number of stars used in each comparison in Table III is indicated by the figures in parentheses.

It is obvious from the method of derivation that the mean values in Table III for all the pairs of lines will be small in the case

TABLE III

Ratio	F8 to G6			G6 to K9		
	$M=-2.9$	$M=+6.1$	Remarks	$M=-2.7$	$M=+6.2$	Remarks
$\frac{4227}{H_\gamma}$	−0.2 (19)	−0.1 (47)	Class.	+1.9 (16)	+0.5 (19)	
$\frac{4326}{H_\gamma}$	0.0 (28)	0.0 (49)	Class.	−0.2 (52)	0.0 (45)	Class. (wt. ½)
$\frac{H_\gamma}{4352}$	−0.3 (28)	+0.1 (49)	Class.	+0.9 (52)	0.0 (45)	Class.
$\frac{H_\gamma}{4405}$	+0.4 (28)	−0.1 (49)	Class.	+0.1 (52)	0.0 (45)	Class. (wt. ½)
$\frac{H_\gamma}{4384}$	+0.6 (28)	0.0 (48)	Class.	+0.2 (51)	+0.2 (43)	
$\frac{H_\beta}{4852}$	−0.7 (13)	+0.2 (29)		−0.4 (48)	−0.1 (43)	Class.
$\frac{H_\beta}{4958}$	+0.1 (16)	−0.2 (28)		−0.9 (46)	0.0 (40)	Class.
$\frac{4337}{H_\gamma}$	+3.2 (28)	−0.4 (49)		+1.9 (52)	+0.3 (45)	
$\frac{4415}{4435}$	−1.2 (28)	−0.2 (49)		−1.1 (52)	+0.1 (45)	
$\frac{4435}{4444}$	−1.0 (28)	−0.1 (49)		−0.7 (52)	−0.2 (45)	
$\frac{4415}{4456}$	+1.7 (28)	−0.1 (49)		+3.9 (52)	+0.2 (45)	
$\frac{4456}{4462}$	−4.8 (28)	+0.4 (49)	Abs. mag.	−6.6 (52)	−0.3 (45)	Abs. mag.
$\frac{4456}{4495}$	−3.6 (26)	+0.5 (36)	Abs. mag.	−5.2 (51)	−0.3 (45)	Abs. mag.
$\frac{4216}{4250}$	+8.3 (21)	−0.4 (44)	Abs. mag.	+5.8 (31)	+0.2 (33)	Abs. mag.
$\frac{4250}{4255}$	−0.5 (20)	−0.1 (45)		−1.3 (42)	−0.1 (38)	
$\frac{4271}{4275}$	−1.2 (27)	0.0 (46)		−0.5 (48)	−0.3 (43)	
$\frac{4368}{4376}$	−0.8 (28)	−0.3 (41)		+0.1 (52)	0.0 (41)	
$\frac{4395}{4415}$	+4.1 (28)	−0.2 (41)	Abs. mag.	+3.4 (52)	+0.4 (40)	Abs. mag.
$\frac{4408}{4415}$	+5.7 (28)	+0.1 (48)	Abs. mag.	+4.9 (52)	+0.5 (45)	Abs. mag.
$\frac{4495}{4497}$	−3.4 (15)	−1.0 (4)		−2.8 (46)	−0.9 (14)	
$\frac{4462}{4495}$	+1.2 (25)	+0.1 (33)		+1.7 (51)	+0.2 (45)	
$\frac{4415}{4535}$	−2.3 (27)	0.0 (40)		−1.5 (52)	0.0 (45)	
$\frac{4325}{4326}$	−4.0 (21)	+0.1 (46)		−5.1 (27)	+0.2 (37)	

of the stars of small absolute magnitude, and that the values for the pairs used for classification purposes will be small for stars of both small and large absolute magnitude. The most prominent cases of lines where systematic differences are seen to exist between the stars of high and of low luminosity are the following:

STARS OF HIGH LUMINOSITY

Strong	Weak
	4325 Sc II
4216 Sr II	4435 Ca
4395 Ti, V, Zr	4456 Ca
4408 V, Fe	4535 Ti

The Sr line at λ 4216 is an extremely prominent chromospheric line, and the same is true in less degree of the enhanced Ti line at λ 4395. The line at λ 4408 is a blend, and as given by Rowland consists of V and Fe. Some other element may perhaps contribute to the stellar line. All four of the lines which are relatively weak in the high luminosity stars are well known sun-spot lines, being greatly strengthened in the umbrae of spots.

The following five pairs of lines were selected from Table III as the basis for an investigation of the individual stars:

4216	4395	4408	4456	4456
4250	4415	4415	4462	4495

The results given in Table III, estimated value−normal value, for these five pairs of lines were combined into means. By assuming a linear relationship between these mean values D, and the absolute magnitude M, we then derived the formulae:

F8–G6 stars: $M = +5.6 - 1.6\,D$
G6–K9 stars: $M = +6.8 - 1.8\,D$

The difference between the two constant terms shows merely that the average magnitude of the stars used for the normal curve is 5.6 for the first group, and 6.8 for the second group. The agreement for the two groups of the coefficient of D indicates how well the same relationship holds throughout the whole range of spectral type from F8 to K9. For the very faintest stars, below absolute magnitude 7, the linear relationship does not seem to hold strictly but it has not seemed desirable for the present material to use a more complicated formula.

Tables IV and V show the absolute magnitudes computed from these formulae for 71 stars of types F8 to G6, and 91 stars of types G6 to K9. The spectral classification is that derived by the method already described and the parallax π is taken from *Groningen Publication*, No. 24. The first column of absolute magnitudes M contains the values calculated from the parallax or the proper motion, the latter being used wherever the measured parallax is less than $+0''.05$. The second column of absolute magnitudes contains the values determined from the intensities of the spectrum lines.

The average difference between the two sets of absolute magnitudes is slightly less than 1.6 magnitudes for the F8–G6 stars, and 1.5 magnitudes for the G6–K9 stars. In view of the uncertainties attaching to the determination of absolute magnitudes from proper motions, this difference is not excessive. There appears, therefore, to be considerable promise in the application of spectrum line criteria to the determination of absolute magnitudes and parallaxes.

SUMMARY

Including the results described here, we have found as a product of our investigations of the spectra of large and of small proper motion stars three phenomena which appear to have a distinct bearing upon the problem of the determination of the absolute magnitudes of stars.

1. The continuous spectrum of the small proper motion stars is decidedly less intense in the violet region relative to the red than the spectrum of the nearer and smaller stars. This effect appears to be a function of the spectral type, and so must be ascribed in part, at least, to conditions in the stellar atmospheres.

2. A considerable number of the small proper motion stars show hydrogen lines of abnormally great intensity. Measures of the radial velocity show the source of the additional absorption to be mainly, if not wholly, in the stars themselves.

3. Certain lines are strong in the spectra of the small proper motion stars, and others in the spectra of the large proper motion stars. The use of the relative intensities of these lines gives results

TABLE IV

F8–G6

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Brad. 3212	6.2	G6	0".419	+0".15	+7	+ 7
Boss 9	6.3	G4	0.006	.....	−3	− 3
Boss 41	5.8	G2	0.002	.....	−6	− 1
Pi. 0^h130	5.7	G5	1.390	+ .36	+8	+ 6
Pi. 0^h137	7.4	G1	0.790	+ .02	+7	+ 6
Boss 138	5.4	G6	0.007	.....	−4	− 2
μ Cassiop.	5.3	G3	3.760	+ .11	+6	+10
Lal. 2450	8.1	G0	0.570	+ .02	+7	+ 8
Boss 349	5.5	G5	0.008	.....	−4	− 2
Boss 355	5.7	G6	0.004	.....	−5	− 5
Fed. 263	7.6	G4	0.750	+ .04	+7	+ 9
Lal. 3922–3	7.5	G0	0.510	− .04	+6	+ 4
Lal. 4141	6.9	G4	0.600	+ .03	+6	+ 6
Boss 619	5.6	F9	0.004	.....	−5	− 1
Lal. 5490–6	6.7	G3	1.000	+ .06	+5	+ 6
Boss 977	5.6	G6	0.020	.....	−2	− 6
Groom. 864	7.3	G0	0.730	+ .03	+6	+ 8
Boss 1131	6.8	G1	0.440	.....	+5	+ 7
Groom. 884	7.1	F9	0.680	+ .09	+7	+ 7
Lal. 9091	7.0	F8	0.100	− .02	+3	+ 6
α Aurigae	0.2	G1	0.437	+ .07	−1	+ 1
λ Aurigae	4.8	G1	0.843	+ .11	+5	+ 7
Boss 1394	6.4	G5	0.002	.....	−5	− 1
Boss 1441	5.9	G4	0.014	.....	−2	− 1
Lal. 11196	6.5	G2	0.690	+ .06	+5	+ 5
Boss 1513	5.8	G5	0.007	.....	−4	− 4
Boss 1704	6.5	G1	0.017	.....	−1	− 1
W.B. 6^h1500	7.7	G3	0.570	+ .01	+6	+ 7
Lal. 13849	6.5	G6	0.520	+ .11	+7	+ 6
Boss 1873	6.5	G4	0.019	.....	−1	− 2
28 Hev. Cam	6.5	G6	0.478	.00	+5	+ 5
Lal. 15565	6.9	G7	1.190	+ .09	+7	+ 7
Boss 2150	5.5	G3	0.006	.....	−4	− 1
Pi. 7^h321	7.0	G3	0.807	+ .05	+5	+ 6
Boss 2236	6.3	G1	0.362	.....	+4	+ 7
Lal. 16904	8.1	G3	0.440	− .04	+6	+ 6
Boss 2338	6.1	G3	0.020	.....	−1	− 1
Lal. 19896	7.7	G0	0.470	− .95	+6	+ 5
Pi. 11^h32	7.3	G6	0.808	+ .01	+7	+ 8
Groom. 1822	7.9	G0	0.670	+ .02	+7	+ 2
Lal. 22585	6.4	G6	0.470	+ .12	+7	+ 4
Groom. 1855	7.4	G6	0.340	+ .06	+6	+ 8
W.B. 12^h69	7.3	G4	0.740	.00	+6	+ 5
Lal. 22908	7.5	G3	0.590	+ .09	+7	+ 6
Lal. 24414–6	6.9	G3	0.720	+ .01	+6	+ 5
Boss 3442	6.3	G6	0.008	.....	−3	− 1
Pi. 13^h114	7.5	G6	0.940	+ .05	+6	+ 7
Lal. 27742 Br	6.8	G4	0.680	.00	+6	+ 4
Lal. 27742 Ft	7.6	G6	0.680	.00	+7	+ 7
Lal. 30694	6.8	G6	1.660	+ .07	+6	+ 7
Lal. 31132	6.7	G5	0.830	+0.14	+7	+ 7
Boss 4601	5.9	G6	0.009	.....	−3	− 2

TABLE IV—*Continued*

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Boss 4763	6.2	G5	0".010	.....	−3	− 2
Groom. 2789N	6.6	G3	0.654	+0".02	+6	+ 7
Groom. 2789S	6.8	G2	0.630	+ .02	+6	+ 6
Boss 4910	6.3	G5	0.014	.....	−2	+ 1
16 Cygni Pr	6.3	G2	0.212	+ .16	+7	+ 9
16 Cygni Fol	6.4	G2	0.220	+ .16	+7	+ 3
Lal. 38287	7.2	G6	0.600	+ .09	+7	+ 8
Pi. 20^h174	5.8	G2	0.317	+ .04	+3	+ 4
Groom. 3215	7.0	G6	0.480	+ .01	+5	+ 6
Fed. 3638	7.8	G4	0.700	− .02	+7	+ 6
Boss 5420	6.2	G6	0.012	.....	−2	− 1
Lac. 8777	6.5	G5	0.680	− .10	+6	+ 4
Lal. 43492	6.9	F9	0.840	+ .02	+6	+ 8
Boss 5769	6.1	G5	0.006	.....	−4	0
Fed. 4371	7.5	G3	0.600	+ .06	+6	+ 8
Boss 5982	6.5	Go	0.015	.....	−1	− 1
B.D. 62°2244	7.3	G4	0.440	− .02	+6	+ 4
Pi. 23^h164	7.0	F8	0.620	− .05	+6	+ 7
Pi. 23^h267	6.2	Go	0.770	+0.01	+5	+ 1

for absolute magnitudes in satisfactory agreement with those derived from parallaxes and proper motions.

It seems very probable from physical considerations that the spectra of stars of quite different mass and size would differ considerably in certain respects even when the main spectral characteristics were the same. If the depth of the atmosphere for stars of similar spectral type is at all in proportion to the linear dimensions of the stars, we should expect the deeper reversing layers of the larger stars to produce certain modifications of the spectrum lines. Owing to the small scale of the stellar spectrum photographs, only the most marked changes could be distinguished, and among these the effect of the deep atmosphere upon the violet end of the spectrum should be especially prominent.

A case of somewhat similar nature is that found in observations of the center and the limb of the sun. The length of path through the solar atmosphere is much greater at the limb, and greater relatively for the lower and lower strata. On large-scale solar photographs the differences between the center and the limb spectra are very marked, but on very small-scale photographs, no doubt, only the most prominent differences could be observed.

TABLE V

G6–K9

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Boss 131	5.7	K0	0″.019	.....	−2	− 2
54 Piscium	6.1	K1	0.595	+0″.14	+7	+ 5
Lal. 1045	7.5	K3	0.810	+ .02	+7	+ 8
Boss 165	6.2	G6	0.016	.....	−2	− 3
Mayer 20	5.8	K1	1.366	+ .17	+7	+10
Boss 209	5.6	G6	0.017	.....	−2	− 6
Lal. 1799	8.0	K6	0.480	+ .06	+7	+ 6
Boss 267	6.2	K1	0.004	.....	−4	− 4
W.B. 1^h161	8.0	G8	0.490	− .01	+6	+ 8
Lal. 2682	7.8	K1	0.470	+ .01	+6	+ 5
Lal. 3022	7.8	G7	0.500	+ .05	+6	+ 9
Boss 410	5.9	G9	0.018	.....	−2	− 3
Boss 420	5.6	K1	0.009	.....	−3	− 4
Boss 430	6.1	K1	0.005	.....	−4	− 3
Boss 432	5.8	K0	0.180	.....	+2	− 1
Boss 434	6.0	G8	0.011	.....	−3	− 2
α Arietis	2.2	G8	0.239	+ .09	+2	− 2
Boss 493	6.2	K4	0.011	.....	−2	− 2
Boss 539	5.6	G7	0.006	.....	−4	− 3
Boss 581	5.9	G9	0.010	.....	−3	− 3
Pi. 2^h123	5.9	K6	2.320	+ .14	+7	+ 7
W.B. 2^h927	8.2	G8	0.680	+ .10	+8	+ 8
W.B. 3^h113	7.8	K0	0.620	+ .08	+7	+ 1
Boss 768	5.6	G8	0.020	.....	−2	− 4
Boss 832	6.0	G7	0.002	.....	−6	0
γ Tauri	3.9	G9	0.120	.....	0	− 1
Boss 1014	6.1	G7	0.011	.....	−2	− 2
ϵ Tauri	3.6	K0	0.120	.....	−1	0
θ Tauri	4.0	G8	0.108	.....	+1	+ 1
α Tauri	1.1	K3	0.203	+ .07	0	− 1
Boss 1176	6.4	K2	0.012	.....	−2	− 2
W.B. 4^h1189	6.5	K9	1.250	+ .30	+9	+ 5
Boss 1247	5.8	K1	0.017	.....	−2	− 2
Boss 1348	6.0	K0	0.007	.....	−4	− 4
Groom. 990	8.1	K0	0.560	+ .03	+7	+11
Pi. 5^h146	6.4	K0	0.510	+ .12	+7	+ 7
Lal. 10797–8	7.3	K2	0.720	+ .08	+7	+ 7
Boss 1444	5.6	G8	0.009	.....	−3	− 4
Boss 1608	5.5	K2	0.011	.....	−3	− 2
6 Lyncis	5.9	G7	0.330	− .01	+4	+ 4
Boss 1632	6.0	K2	0.007	.....	−4	− 2
Boss 1643	5.9	K6	0.019	.....	−2	− 3
Lal. 13284–5	6.9	K9	0.570	+ .11	+7	+ 6
Lal. 13427	8.2	K1	0.710	+ .05	+7	+ 7
Boss 1846	5.7	K0	0.019	.....	−2	− 4
Boss 1868	5.9	K2	0.020	.....	−2	− 2
Boss 2148	6.4	K2	0.073	− .04	+1	− 2
Boss 2220	6.2	G8	0.011	.....	−2	− 1
Pi. 8^h105	6.0	K3	0.040	−0.03	0	− 1
55 ρ^1 Cancri	6.1	K0	0.541	+ .08	+6	+ 2
Boss 2449	5.2	G6	0.008	.....	−4	+ 1
Boss 2455	6.1	G6	0.010	.....	−3	− 2

TABLE V—*Continued*

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Lal. 18286	7.3	K3	0".520	+0".07	+6	+ 8
Lal. 19022	8.2	K5	0.800	+ .06	+7	+ 6
Groom. 1596	8.2	G6	0.480	+ .07	+7	+ 6
Boss 2795	6.0	G7	0.016	.....	−2	− 2
Boss 2910	5.2	K2	0.113	+ .02	+1	− 2
83 Leonis Br.	6.2	G8	0.743	+ .02	+6	+ 6
83 Leonis Ft.	7.6	K8	0.736	+ .02	+7	+ 5
Boss 3125	5.9	K1	0.012	.....	−3	− 2
Pi. 11^h218	6.8	G6	0.670	− .03	+6	+ 2
Boss 3406	6.0	G9	0.020	.....	−2	− 3
Lal. 26196	7.6	K1	0.680	+ .14	+8	+ 7
Boss 3810	5.6	G9	0.002	.....	−6	− 2
Lal. 27744	6.7	G7	1.380	+ .13	+7	+ 8
OΣ 298	7.9	K6	0.480	+ .05	+6	+ 7
W.B. 15^h720	6.8	K1	0.480	+ .05	+5	+ 6
Lal. 30024–6	7.0	K1	0.490	+ .09	+7	+ 9
Boss 4228	6.0	G8	0.020	.....	−2	− 6
Lal. 30699	7.8	G7	0.470	+ .05	+6	+ 4
Lal. 33439	6.7	K2	0.650	+ .04	+6	+ 8
Boss 4724	5.8	G8	0.006	.....	−4	0
Groom. 2875	6.7	K1	0.660	− .05	+6	+ 8
Lal. 38383	7.2	K1	1.390	+ .03	+8	+10
Pi. 20^h23	7.3	G9	0.550	+ .08	+7	+ 9
Boss 5177	5.8	G8	0.020	.....	−2	− 3
Pi. 20^h29	5.7	G9	1.256	+ .01	+6	+ 6
Boss 5317	6.2	K1	0.012	.....	−2	− 2
η Cephei	3.6	K0	0.836	+ .10	+4	+ 3
Boss 5397	5.6	K4	0.005	.....	−4	− 4
Boss 5486	6.2	K0	0.020	.....	−1	− 2
Boss 5655	5.8	K1	0.006	.....	−4	− 2
Groom. 3689	8.1	G8	0.610	+ .04	+7	+ 6
Boss 5868	6.1	K4	0.016	.....	−2	− 4
Pi. 22^h214	6.5	G9	0.470	− .02	+5	+ 3
Lal. 45028	7.8	K2	0.500	+ .05	+6	+ 7
Brad. 3077	5.6	K7	2.105	+ .16	+7	+ 5
Lal. 45755	7.6	G8	0.670	+ .04	+7	+10
Boss 6016	5.9	G8	0.300	.....	−5	− 3
B.D. $+58°2605$	7.5	K1	1.080	+0.07	+7	+ 8
Boss 6123	5.8	K0	0.020	.....	−2	− 2
Boss 6176	5.8	G7	0.018	.....	−2	− 2

The difference, however, in the relative intensity of the violet portion of the continuous spectrum at center and limb as compared with the red portion, which is so marked a feature of the observations, would appear equally well on photographs taken with high and low dispersion.

MOUNT WILSON SOLAR OBSERVATORY
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THE SPECTROSCOPIC ORBIT OF RX HERCULIS DETERMINED FROM THREE PLATES WITH A NEW PHOTOMETRIC ORBIT AND ABSOLUTE DIMENSIONS[1]

By HARLOW SHAPLEY

The ordinary solution for the orbit of a spectroscopic binary involves six independent unknowns. These elements may be the period, an epoch, the longitude of periastron, the eccentricity of the orbit, the maximum apparent orbital velocity, and the radial velocity of the system with respect to the sun. Frequently the number of unknowns is reduced to four by the assumption of a circular orbit.

The complete solution for the elements of an eclipsing binary from its light-curve may involve as many as thirteen independent unknowns, but the precision of the photometric observations is seldom sufficient to permit the derivation of more than seven or eight of the quantities theoretically possible. The elements most generally obtained are the period, an epoch, the inclination, the relative light-emissions and dimensions of the two stars, the radius of the orbit, and occasionally the eccentricity and longitude of periastron.[2]

Solutions for orbits of double stars based upon variations in radial velocity, therefore, give four elements in common with solutions based on variations in apparent magnitude, and it is obvious that for any star which is both an eclipsing and a spectroscopic binary the existence of either solution greatly simplifies the derivation of the other, both from an observational and from a computational point of view. There is accordingly a particular

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 90.

[2] For some systems it is possible to determine also the ellipticity of the stars, the reflection effect, and the periastron effect, and to estimate the degree of darkening at the limb (*Astrophysical Journal*, **39**, 405, 1914). The apparent stellar magnitude at maximum is a thirteenth unknown, but except in systems of elliptical stars its determination is usually independent of the study of the light-curve.

interest and advantage in making photometric observations of certain spectroscopic binaries whose orbits are known,[1] and likewise in making spectroscopic observations of eclipsing binaries for which good light-curves exist or are possible; for not only is the problem of the second orbit in either case much simplified, but also the combination of the two sets of computed data affords knowledge of the absolute radii, masses, and densities of the component stars, and occasionally of the surface luminosities in terms of the sun.

Suppose the complete solution of a light-curve has given, among other quantities, four of the six unknowns generally derived from spectroscopic observations: P, the period; T, the epoch of principal minimum (or of node or periastron passage); e, the eccentricity; and ω, the longitude of periastron counted from the ascending node of the primary component. Then each determination of the radial velocity of the brighter star gives the linear relation

$$V_1=\gamma+K_1[\cos u+e\cos\omega] \tag{1}$$

between the two remaining unknowns—γ, the constant radial velocity of the center of mass of the system, and K_1, the semi-amplitude of the velocity variation. (u, the argument of the latitude, is readily obtained from tables when P, e, and ω are known.) If the lines in the spectrum of the second component are measurable, then

$$V_2=\gamma+K_2\,[\cos u+e\cos(\omega+\pi)] \tag{2}$$

and in both cases the last term in the brackets drops out for orbits circular or nearly so. We have then simply

$$V=\gamma+K\sin\theta \tag{3}$$

where θ is the phase angle from minimum. It is evident that a very few spectrograms made near greatest elongation, giving a large coefficient of K, will suffice, whether the orbit is circular or eccentric, to obtain γ and K with average precision, hence to determine completely the spectroscopic orbit. The following discussion shows what use we can make of the measures of a few lines on three plates

[1] See discussion of this phase of the combination by Stebbins, *Astrophysical Journal*, **34**, 105, 1911.

of RX Herculis, an eclipsing binary the solution of whose light-curve eliminates four unknowns from the spectroscopic problem.

THE PHOTOMETRIC OBSERVATIONS

A new light-curve of RX Herculis is derived below from two series of observations, one made at Harvard and one at Princeton, neither of which has as yet been published. A long series of visual estimates of this star was obtained by Luizet during the years 1899–1904.[1] Especial care had been taken in making the observations, with the result that the light-elements derived from them are very accurate. The light-scale is not photometric, however, and it is not advisable to put great reliance upon the range of variation or upon the shape of the curve. The period given by Luizet is $0^d.889288$, and the range of variation is $0^m.63$, with the maximum at the seventh magnitude, approximately. The maximum light had been observed more than 300 times and there was no evidence of ellipticity or of a secondary minimum.[2]

The preliminary solution of Luizet's light-curve by the present writer was indeterminate. It was found that either the period must be double the given value with primary and secondary eclipses indistinguishable, or the period must be as given and the system supposed to consist of a small bright star with a faintly luminous, larger companion. The shape of the light-curve as derived from Luizet's measures contradicted the first supposition and the absence of a shallow secondary minimum contradicted the second. As the star is bright enough for spectroscopic work, it was considered of sufficient importance to justify a thorough photometric study, and accordingly it was put on the list of stars to be observed by the writer with the polarizing photometer of the Princeton University Observatory. The observations, which are published in detail elsewhere, are discussed briefly in the following paragraphs.

[1] The observations and light-curve are published in *Astronomische Nachrichten*, **168**, 283, 1905, and *Bulletin astronomique*, **22**, 232, 1905; see also *Astronomische Nachrichten*, **165**, 183, 1904; *Astronomical Journal*, **22**, 162, 1902; *Popular Astronomy*, **21**, 142, 1913.

[2] In conspicuous disagreement are the results deduced by Yendell from visual estimates, *Astronomical Journal*, **22**, 162, 1902.

The comparison star used in observing RX Herculis is a = B.D. $+12°3546$ ($7^m.7$). The observations were unusually difficult on account of the brightness of the two stars, and because many of the measures were made when the field was very low. Of the total of 133 sets (each of 16 comparisons) 63 were made during changing light. The 70 sets outside of eclipse, in agreement with Luizet's results, show no deviation from constant light. Hence the shorter period, which demands a secondary at the phase of 0.445 days, becomes doubtful. Moreover, an examination of the observations during changing light shows that the branches are not so steep as measured by Luizet and, therefore, that the double-period solution for the orbit is now suitable.

An investigation of the depths of alternate minima did not reveal definitely, so far as my observations were concerned, any difference larger than may be accounted for by errors of observation, but the data bearing on the point were rather weak. There was an indication that the minima at even epochs were slightly deeper, but it was not until the Harvard observations were examined that the difference was established beyond doubt. Consequently the photometric orbit already published is based upon an assumption of eclipses equally deep.[1] The existence of unequal minima shows at once that only one-half a revolution period elapses between successive eclipses. A third point in favor of the double value of the period is the spectroscopic evidence obtained at the Yerkes Observatory. The lines due to both components are visible on the plates made in 1905.[2] The recent measurement of these plates has furnished the data upon which the present spectroscopic orbit is based. They will be discussed on a later page.

A new mean epoch of heliocentric minimum, derived from the curve of the several partially observed eclipses, is J.D. 2419658.5882, G.M.T. This time is eight minutes in advance of that predicted and, assuming that no secondary oscillations exist, furnishes a small correction to Luizet's period. The new light-elements are:

$$\text{Primary Minimum} = \text{J.D. } 2419658.5882 + 1^d.7785740\ E.$$

[1] *Astrophysical Journal*, **38**, 163, 1913.

[2] *Ibid.*, **22**, 215, 1905.

The weighted mean of the maximum light measures is $a-v=0^m.877 \pm 0^m.004$, the probable error of a single observation being $\pm 0^m.033$.

Through the kindness of Professor Pickering the proof sheets of a series of 127 observations of RX Herculis, made at Harvard by Professor Wendell during the years 1906–1910, have been furnished me in advance of publication in *Harvard Annals*, **69**, Part II. The measures were made with a polarizing photometer identical with the one used at Princeton, and as each set consists also of sixteen comparisons, the combination with equal weight of the observations of the two series is entirely justifiable. The maximum light, according to 23 observations by Wendell, is $7^m.07 \pm 0^m.006$, and the probable error of a single observation is slightly less than $\pm 0^m.03$. For combination with the Harvard results all the Princeton magnitudes were reduced so that the maximum magnitude should have the above value.

The preliminary plot of Wendell's observations confirmed the correction to Luizet's light-elements, but in addition showed an outstanding displacement of the minimum. This latter difficulty was found traceable to the chance adoption without alteration of the originally published initial epoch, which was referred to the Paris meridian, while the observation times are all referred to Greenwich. Two positive corrections have been applied, therefore, to the phases published at Harvard: one of ten minutes to reduce to Greenwich time, and the other increasing with the time from four to seven minutes to allow for the revision of the period.

With these adjustments the two sets of observations are found to be in extremely good agreement. The difference in the depths of successive minima of nearly a tenth of a magnitude is shown distinctly. In primary minimum there are 38 observations from Harvard and 41 from Princeton; in secondary, 66 observations from Harvard and 25 from Princeton. In the following tables of normal magnitudes the observations in the two minima are collected in order of phase into groups of five.[1] The phases refer to the middle epochs of the two minima. The third and fourth

[1] Half-weight is given to two Princeton observations made under very unfavorable conditions. The phase of one Harvard observation is arbitrarily altered because of the evident impossibility of the recorded time of observation.

columns give the number of observations contributed from Harvard and Princeton, respectively, to each normal. The last two columns contain the residuals from theoretical light-curves computed on the

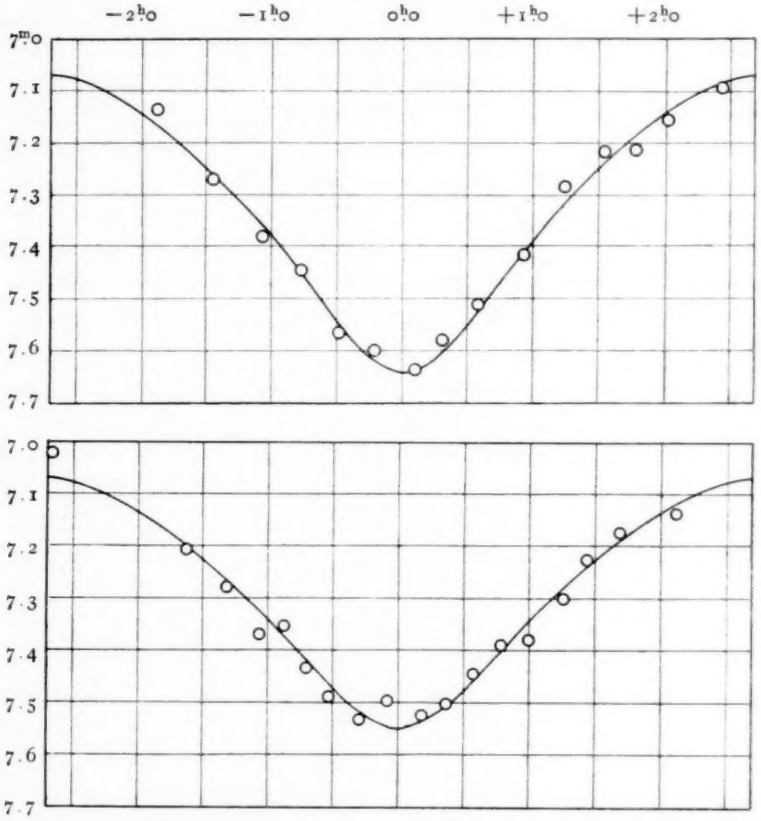

FIG. 1.—The mean light-curve of RX Herculis; primary minimum above, secondary minimum below.

two hypotheses of uniformly luminous disks and of disks completely darkened at the edge. In Fig. 1 are given the normal points and the computed uniform light-curves, with the primary minimum at the top of the diagram.

THE SPECTROSCOPIC OBSERVATIONS

The total available spectroscopic data for RX Herculis consist of four spectrograms, made with the Bruce spectrograph at the

Yerkes Observatory. Because of the faintness of the star and the nature of its spectrum, the investigation of its orbit has not hitherto

TABLE I

NORMAL MAGNITUDES OF RX HERCULIS NEAR PRIMARY MINIMUM

No.	Phase	No. Observations		Magnitude	O−C	
		Harvard	Princeton		Uniform	Darkened
1	-1^h53^m0	3	2	7.136	-0^m03	-0^m01
2	−1 27.1	0	5	7.266	+ .01	+ .02
3	−1 4.6	2	3	7.380	+ .01	+ .01
4	−0 47.4	2	3	7.442	− .01	− .02
5	−0 29.8	3	2	7.564	+ .02	+ .01
6	−0 13.2	2	3	7.600	− .02	.00
7	+0 5.0	3	2	7.636	.00	+ .02
8	+0 18.0	2	3	7.578	− .03	− .02
9	+0 34.8	2	3	7.512	− .01	− .02
10	+0 55.4	3	2	7.414	.00	.00
11	+1 14.4	3	2	7.284	− .03	− .04
12	+1 32.0	4	1	7.216	− .03	− .01
13	+1 46.8	3	2	7.212	+ .03	+ .04
14	+2 0.6	2	3	7.154	+ .01	+ .03
15	+2 26.2	3	2	7.096	+ .01	+ .02
16	+2 58.5	1	3	7.058	−0.03	−0.03

TABLE II

NORMAL MAGNITUDES OF RX HERCULIS NEAR SECONDARY MINIMUM

No.	Phase	No. Observations		Magnitude	O−C	
		Harvard	Princeton		Uniform	Darkened
1	-2^h40^m0	0	5	7.022	-0^m05	-0^m05
2	−1 37.8	3	2	7.206	+ .01	+ .03
3	−1 18.4	4	1	7.280	+ .01	+ .02
4	−1 3.8	4	1	7.370	+ .04	+ .03
5	−0 53.6	5	0	7.352	− .02	− .03
6	−0 42.8	3	2	7.436	+ .02	+ .01
7	−0 31.8	4	1	7.490	+ .02	+ .01
8	−0 18.0	4	1	7.536	+ .02	.00
9	−0 5.6	4	1	7.498	− .05	− .06
10	+0 10.8	4	1	7.522	− .01	− .03
11	+0 21.8	4	1	7.502	.00	− .02
12	+0 34.4	3	2	7.444	− .02	− .03
13	+0 47.2	4	1	7.394	− .01	− .02
14	+0 59.8	3	2	7.380	+ .03	+ .03
15	+1 14.8	4	1	7.302	+ .02	+ .03
16	+1 26.6	3	2	7.226	− .01	.00
17	+1 41.2	4	1	7.174	− .01	.00
18	+2 6.3	6	0	7.138	+0.02	+0.03

been attempted, and only a short note has been published in general description of the spectral lines.[1] At my request Professor Frost very kindly consented to measure the plates, as far as might be possible, and has recently communicated to me his results. Upon one of the plates the lines were too indefinite to permit a quantitative estimate of the velocity. For the other three the results are as tabulated in Table III. Concerning these measures Professor Frost writes:

For the first two of the plates the results are clear, and have some fair degree of reliability, but I would not affirm that they were nearer than 10 km of the truth. . . . . I have re-examined the plates with a view of distinguishing between the components in intensity. The slightly less intensity of the broader component makes estimation difficult. On Plate 558 at λ 4481, broader component (toward red) showed slightly fainter than narrower component. The same remark refers to Plate 562. On Plate 839 the width and intensity of the components at λ 4481 were thought to be equal and the same for the other lines on that plate.

TABLE III

Plate No.	Date	G.M.T.	Primary Component		Secondary Component	
			Velocity	No. Lines	Velocity	No. Lines
558.....	1905, July 16	16^h28^m	+ 84 km	2	−130 km	2
562.....	1905, July 21	17 1	+ 42	3	− 74	4
839.....	1906, Sept. 8	15 25	−108	3	+ 75	2

On plate 839 the measure of one component of the line at λ 4481 gave a velocity of +188 km. The note is added:

I cannot well account for the discrepancies in the positive values, but the very large positive displacement at λ 4481 is evident. The two other positive lines gave values of +69 and +82 km.

As this large positive value is in glaring disagreement with all the other measures on all the plates, there is nothing to do but neglect it completely.

THE PHOTOMETRIC ORBIT

Although a fairly good photometric orbit of RX Herculis is already available,[2] it was thought best to revise the solution because

[1] *Astrophysical Journal*, **22**, 215, 1905.

[2] *Ibid.*, **38**, 158, 1913.

of the additional photometric material from Harvard. The computations have followed the usual lines,[1] except that in certain details the spectroscopic data have influenced the choice between possible, ambiguous interpretations of the light-variations. The revised light-curve has shown definitely that the minimum at the even epoch is the deeper. The star approaching after that eclipse, according to the estimates of Professor Frost, is probably somewhat the fainter. At first glance there may appear to be a contradiction here. In general at the deeper minimum the star of greater surface intensity is eclipsed,[2] and almost invariably in practice the component with greater surface brightness has the greater total brightness.[3] If, however, the more intense star is small enough relative to its companion, it may have less than half the light and accordingly show fainter spectral lines. It is found from a study of the light-curve that RX Herculis is one of these rare exceptions and that the condition—fainter component smaller and with greater surface intensity—is not only possible but is required by the most probable solution.

Let us consider first the orbit from the standpoint of uniformly luminous disks. The range of variation at primary minimum is $0^m.57$ and at secondary $0^m.48$, corresponding to losses of light in terms of the total of $1-\lambda_p=0.408$ and $1-\lambda_s=0.357$, respectively.[4] The solution of the light-curve at the two minima gave $\chi\,(k, \alpha_0, \frac{1}{4}) = 2.01$, with an uncertainty of ± 0.01. With the above value of the losses of light this allows an indeterminateness of small range in the values of k and α_0 (the ratio of radii and fraction of eclipse) when computed from the relation $\alpha_0 = 1-\lambda_1+\frac{1-\lambda_2}{k^2}$. If we assume

[1] A detailed discussion of the method of solving for the orbital elements on the basis of two partial eclipses is to be given in *Contributions from the Princeton University Observatory*, No. 3, and an application is there made to the light-curve of RX Herculis.

[2] This does not necessarily hold if the orbit is eccentric and the inclination differs from 90°. For RX Herculis, however, we assume a circular orbit; later this assumption is proved.

[3] There are only three or four exceptions out of a hundred eclipsing binaries.

[4] The notation and formulae used in the present paper are essentially the same as those in former discussions of eclipsing binaries published in the *Astrophysical Journal* during the last two years.

that the large star is in front at primary minimum, k must lie between unity and 0.87; if we assume the small star to be in front at primary minimum, k must lie between unity and 0.98 or between 0.87 and 0.80, and for the last mentioned value the secondary minimum would be grazing total. The foregoing values of k are all possible without misrepresenting the curve; the best value, however, appears to be $k=0.90$ (small star eclipsed at primary) to which corresponds $\alpha_0=0.85$ and $\chi\,(k, \alpha_0, \frac{1}{4})=2.010$.

Suppose that from among the several possible values of the ratio of radii we choose $k=1.00$; then $\alpha_0=1-\lambda_p+1-\lambda_s=0.765$. The light of the star eclipsed at primary is then 53.3 per cent of the total, and the component approaching after primary eclipse would be fifteen-hundredths of a magnitude brighter than its companion, rather than fainter as indicated by the spectrograms. Obviously, then, the star in front at the primary eclipse must be the larger. If $k=r_2/r_1=0.936$, the components would be just equal in brightness. For the best values of the last paragraph ($k=0.90$, $\alpha_0=0.85$), however, the condition is attained that is in complete accordance with the light-curve and the spectrograms, for then the light of the star eclipsed at primary minimum is 48 per cent of the total; that is, the more intense surface, being the smaller, is approximately a tenth of a magnitude fainter. It is possible also that the difference in breadth of the spectral lines may be attributed to the inequalities of the components.

The solution for the elements on the darkened hypothesis gives results differing but little from those derived on the foregoing assumption. The computations have been much more difficult, however, because for unequal stars the theoretical light-curves at the two minima differ considerably from the uniform, and perceptibly from each other. The ranges of variation at the two minima were altered for this reason to $0^m.55$ and $0^m.49$, respectively. There was again a small indeterminateness in the solution for k and α_0', and therefore the same division of light was assumed as was derived before. For $L_2=0.48$, $L_1=0.52$, the ratio of the stars by chance came out the same. Their dimensions also are but little changed from the uniform values, chiefly because the computed duration of darkened eclipse is only three minutes longer

than for the uniform orbit. There is much less uncertainty in the photometric orbit of this star because of the indecision between the darkened and uniform hypotheses than is usually the case.

Light-curves have been computed for the two eclipses in both cases and the residuals from them are given in the last two columns of Tables I and II. The average deviation during changing light is $\pm 0^m.018$ for the uniform solution and $\pm 0^m.020$ for the darkened. The residuals are slightly smaller in primary than in secondary minimum. The representation of the observations in the minima is approximately of the same accuracy as during maximum light, and no attempt further to improve the accordance of theory and observation is justified. A shift of the epoch of secondary by a minute or two would reduce the average deviation slightly and would break up the apparently systematic nature of the run of residuals, but the adjustment would be within the possible errors of the observations and would not affect the computed elements.

TABLE IV

ELEMENTS OF THE PHOTOMETRIC ORBIT OF RX HERCULIS

	Uniform	Darkened
Ratio of radii, k	0.90	0.90
Fraction of eclipse, α_0, α_0',	0.85	0.828
$p(k, \alpha_0)$, $p(k, \alpha_0')$	−0.728	−0.691
$\sin^2 \theta'$	0.1411	0.1476
Semi-duration of eclipse, t'	2^h37^m	2^h40^m
Radius of brighter star, r_b	0.201	0.206
Radius of fainter star, r_f	0.181	0.185
Least apparent distance of centers, $\cos i$	0.069	0.078
Inclination of orbit plane, i	86°.0	85°.5
Light of brighter star, L_b	0.52	(0.52)
Light of fainter star, L_f	0.48	(0.48)
Ratio of surface intensities, $\frac{J_f}{J_b}$	1.14	1.14
"Equal-mass" density		
Bright star, $\bar{\rho}_b$	0.26	0.25
Faintstar, ρ_f	0.36	0.34

The two sets of elements are given in Table IV. The epoch and period have been given on a preceding page. The ellipticity of the stars is negligible according to the maximum light-measures. The eccentricity of the orbit is discussed in the following paragraphs.

DISCUSSION OF ECCENTRICITY

In the solution for the elements of the photometric orbit the apparent equidistance, equal widths, and symmetry of the two minima have been accepted as sufficient proof that the orbit is circular. It may be of interest to see how great an uncertainty there is in this assumption. The two components of the eccentricity, $e \cos \omega$ and $e \sin \omega$, are determined independently of each other from the light-curve, the former with high precision from the displacement of the secondary minimum from the midpoint between the primaries, and the latter indirectly, and with less ease and precision, from the relative durations of the various phases of the two eclipses. Considering first the transverse component,

$$e \cos \omega = \frac{\pi}{P(1+\operatorname{cosec}^2 i)}\left(t_1 - t_2 - \frac{P}{2}\right) = 0.88 \times (\text{disp. in days}) \qquad (4)$$

Inspecting the light-curve, we find that the combined uncertainty in the epochs of the two minima cannot exceed five minutes and is probably less than three. A change of epoch by as much as five minutes would nearly double the average value of the residuals. Hence $e \cos \omega$ is certainly less than ± 0.003.

To determine strictly the uncertainty in assuming $e \sin \omega = 0$, we should make independent solutions of the light-curve of each minimum, using only the depth of the other to make the derivation of the elements complete. Then, with close approximation,

$$e \sin \omega = \frac{r_1'' - r_1'}{r_1'(1.33 + 0.67 \cos \theta')} \qquad (5)$$

where r_1' and r_1'' are the radii of the larger star derived from the solution of the primary and secondary minimum, respectively, and θ' is the phase angle of the end of eclipse. It is sufficient for the present purpose, however, to notice that when the inclination is 90° and $k = 1$ (both of which conditions are approximately fulfilled in this and similar cases), the uncertainty in the radii is one-half that in θ'. Again referring to the light-curve, we find that the uncertainty in the adopted values for the semi-duration of eclipses is less than ± 4 minutes at the primary, and less than ± 5 minutes at the secondary; that is, since $t = P\theta/2\pi$, θ' must lie within

± 0.010 for one eclipse and ± 0.012 for the other of the values adopted. Hence $r_1' - r_1'' < \pm 0.011$, and since the radius of the larger star is approximately 0.20 we conclude from equation (5) that $e \sin \omega < \pm 0.03$. Combining this result with that obtained for $e \cos \omega$, the eccentricity is found to be not larger than 0.03. Its adopted value is zero and the probable error is estimated to be less than ± 0.02.

THE SPECTROSCOPIC ORBIT

The discussion up to this point has disposed of the elements P, T, e, and ω. It remains only to derive γ and K from the velocity variations by means of equation (3), and the spectroscopic orbit is complete. Using the light-elements given on a previous page, the phases of the middle of the exposure times have been computed. They are given in Table V, column 4, expressed in mean longitudes and referred to the preceding principal minimum. The primary star is taken to be the one eclipsed at secondary minimum.

TABLE V

SPECTROSCOPIC OBSERVATIONS OF RX HERCULIS

PLATE NO.	JULIAN DAY AND G.M.T.	EPOCH	PHASE	PRIMARY COMPONENT		SECONDARY COMPONENT	
				Observed Velocity	O−C	Observed Velocity	O−C
558.....	2417043.686	−1473	279°.2	−130 km	−7 km	+ 84 km	−2 km
562.....	2417048.709	−1468	215.8	− 74	+6.5	+ 42	−1.5
839.....	2417462.642	−1235	120.0	+ 75	+1.5	−108	+2.5

Plotting the observed velocities against the phases, we have Fig. 2. A preliminary solution shows that it is possible to assume the components equal in mass and be well within the probable accuracy of the observations. Then $K_1 = K_2$ and we have six equations for the determination of γ and K. Their solution gives:

$$\gamma = -18.5 \pm 1.4 \text{ km}$$
$$K = 106 \pm 1.7 \text{ km}$$

The residuals from the velocities computed with these elements are given in Table V, and, so far as they go, give ± 3.5 km for the probable error of the mean of the measures on one component on

one plate. This accuracy is even more than was hoped for, considering the faintness of the star and the nature of its spectrum. It is to be noted, however, that the plates were purposely made near times of greatest separation of the lines and troubles from blends were thereby avoided.

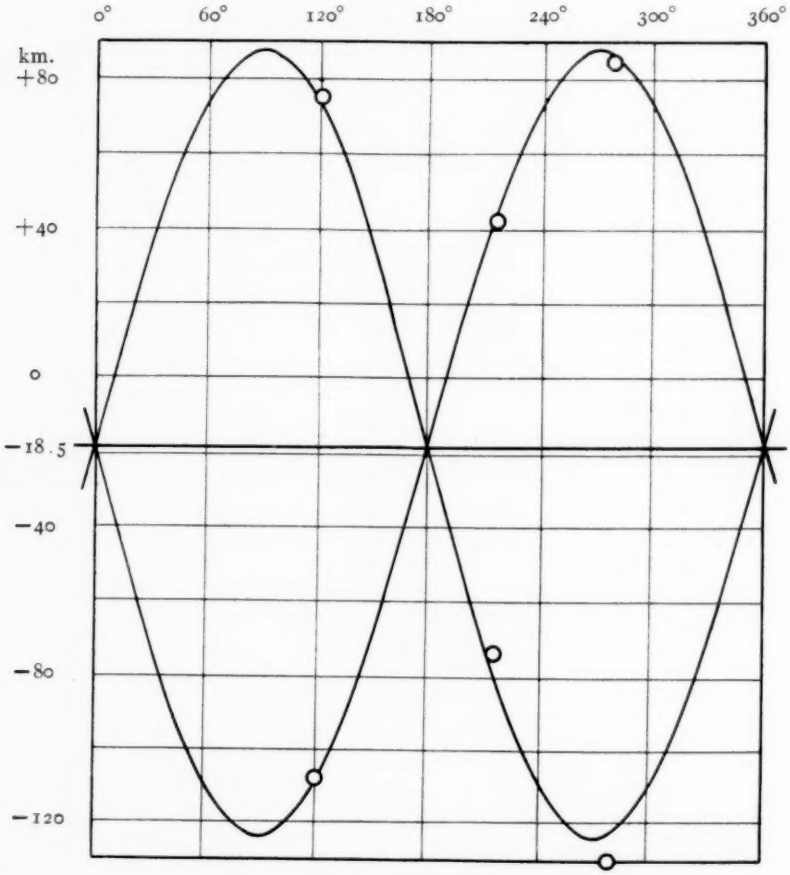

FIG. 2.—The velocity-curve of RX Herculis

It is evident from the velocity-curve that the residuals can still be reduced slightly by assuming the masses not equal. There is an indication that the secondary star, that is, the one with 48 per cent of the light but with the greater surface intensity, is a trifle the more

massive. But as is well known, the fainter companions of all spectroscopic doubles have smaller relative masses. This is without exception, as far as the writer knows. The above condition, then, would be in disagreement with the general rule. But it is at the same time true that, in all cases where such information is available, the star of least surface brightness is the one of smaller mass (for small total light and small surface luminosity go together in general). That would be in agreement with the suggested difference in mass of the components of RX Herculis, but the point is of little importance in this case of nearly equal stars. The masses are without doubt sensibly equivalent and will be so considered.

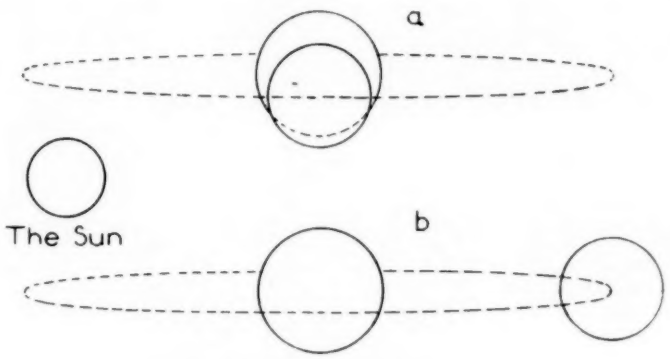

FIG. 3.—The system of RX Herculis compared with the sun
a, at secondary minimum
b, at greatest elongation

The following are the adopted elements of the spectroscopic orbits:

$$P = 1.7785740 \text{ days}$$
$$\mu = 202^{\circ}.4093$$
$$T = \text{J.D. } 2419658.5882 \text{ G.M.T. (primary minimum)}$$
$$e = 0.00 \pm 0.02 \text{ (estimated)}$$
$$K_1 = K_2 = 106 \pm 1.7 \text{ km}$$
$$\gamma = -18.5 \pm 1.4 \text{ km}$$
$$a_1 \sin i = a_2 \sin i = 2{,}590{,}000 \text{ km}$$
$$m_1 \sin^3 i = m_2 \sin^3 i = 0.88$$

The probable errors of the apparent orbital radii and apparent masses are probably less than 2 per cent, but exact computation is not possible since the probable error of the adopted period has not been computed. The star is near the solar apex, and therefore the apparent motion of the center of mass of the system is evidently to be ascribed almost entirely to the motion of the sun. The mass of the system is smaller than we usually find for binaries of early spectral types.

ABSOLUTE DIMENSIONS

The spectroscopic and photometric elements may now be combined and we get the following comparison between RX Herculis and the sun (see Fig. 3):

	Uniform	Darkened
Distance of centers in solar radii	7.47	7.47
Radius of brighter star in solar radii	1.50	1.54
Radius of fainter star in solar radii	1.35	1.38
Apparent distance of centers at middle of eclipse in solar radii	0.52	0.58
Mass of each star	0.89⊙	0.89⊙
Density of brighter star	0.26⊙	0.25⊙
Density of fainter star	0.36⊙	0.34⊙

From the description of the spectrum of RX Herculis in *Astrophysical Journal*, **22**, 215, it should evidently be classed as B9p, the anomalous character being the simultaneous presence of λ 4481 and the faint helium lines. If we knew the relation between the luminosity of its surface and that of a G-type star, we could at once deduce its parallax, since we know the stellar magnitude and the relative surface area of the star with respect to the sun. Or, vice versa, if the parallax were measured directly we could find the relation between the surface luminosities.

Assuming various values of the relative brightness of unit surface areas and expressing this quantity in stellar magnitudes (taking the solar surface as the standard), the computation for the parallax has been made, using the data derived for the brighter component (magnitude 7.78), with the results given in the following table:

	B	A	F	G
Surface luminosity with respect to the sun	$-3^{m}.0$	$-2^{m}.0$	$-1^{m}.0$	$0^{m}.0$
Corresponding brightness per unit surface area	$15.8\odot$	$6.3\odot$	$2.5\odot$	$1.0\odot$
Absolute magnitude	$0^{m}.82$	$1^{m}.82$	$2^{m}.82$	$3^{m}.82$
Parallax	$0''.0041$	$0''.0064$	$0''.0102$	$0''.0162$
Total light of brighter star	$37.3\odot$	$14.9\odot$	$5.9\odot$	$2.4\odot$

The four assumptions with regard to the surface luminosity correspond roughly to what we judge from other sources to be the relative intensities for spectral types B, A, F, and G, respectively. We should expect, then, that the parallax of RX Herculis is of the order of $0''.006$ and that the total light-emission of the system is thirty times that of the sun.

Among the faint eclipsing binaries similar to RX Herculis are the following, for which in a corresponding manner spectroscopic material of considerable value might be obtained without great difficulty:

	Magnitude	Spectrum
U Ophiuchi	5.7	B8
Y Cygni	6.9	A
Z Herculis	7.1	F
RR Centauri	7.4	F
W Ursae Majoris	7.9	G

SUMMARY

1. The solution for the elements of the spectroscopic orbit of a faint star is shown to be easily possible when only a few measures of the radial velocity have been obtained, provided that the system is also an eclipsing binary and provided that the period, the epoch of minimum, the eccentricity, and the longitude of periastron have been derived from the light-curve.

2. A new photometric orbit of RX Herculis has been computed from unpublished observations obtained at Harvard and Princeton (Table IV). Alternate minima are found to differ in depth by nearly a tenth of a magnitude. The stars are nearly equal in size and are sensibly spherical. Their surfaces are separated by three times the radius of the larger star.

3. From measures of the lines on three plates made with the Bruce spectrograph at the Yerkes Observatory, it has been possible to derive very satisfactory spectroscopic orbits of both components.

4. The combination of elements from the photometric and spectroscopic orbits gives the actual dimensions of the stars. (Such information is reliably known for only two other binary systems.) The mass is unusually low for an early-type star. The density is slightly above normal. The volume of both components is six times that of the sun.

5. With a reasonable assumption regarding the surface luminosity, the parallax of the system is found to be $0''.006$. The total light of the system is likely more than thirty times that of the sun.

MOUNT WILSON SOLAR OBSERVATORY
July 26, 1914